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Synchronisation of Linear Agents over a Random Communication Network

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1 Introduction

Modern communication technologies facilitate a flexible exchange of information between autonomous agents via a digital communication network. The control objective of these multi-agent systems is to achieve a coordinated behaviour of the individual agents.

Figure 1 shows the structure of a multi-agent system. The physically uncoupled agents Σ_i, $(i = 1, \dots, N)$, are connected over a networked controller, which consists of local control units C_i, $(i = 1, \dots, N)$, and a communication network $\boldsymbol{L}$.

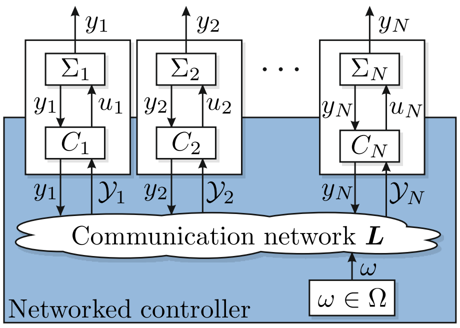

Figure 1: Structure of a multi-agent system

A common control objective is the asymptotic synchronisation of the output signals of the agents

$$\lim_{t \to \infty} |y_i(t) - y_j(t)| = 0, \qquad i, j = 1, \dots, N.$$

For deterministic communication networks, which might be static or vary with time, there exist necessary and sufficient conditions for the design of the networked controller to ensure the asymptotic synchronisation (see [1] for a survey). In contrast, this project focuses on random communication networks. It is assumed that the occurrence of links in the communication network at time t depends on the outcome ω of a random experiment.

2 Communication Network

As depicted in Fig. 2, the random communication network $\vec{\mathcal{G}}(t, \omega) = (\mathcal{V}, \mathcal{E}(t, \omega))$ is modelled by a random sequence of directed graphs $\{\vec{\mathcal{G}}(i, \omega), i \geq 0\}$, each of which is active over the time intervals $[t_i, t_{i+1})$. Any particular random graph $\vec{\mathcal{G}}(i, \omega)$ is obtained independently of the other graphs

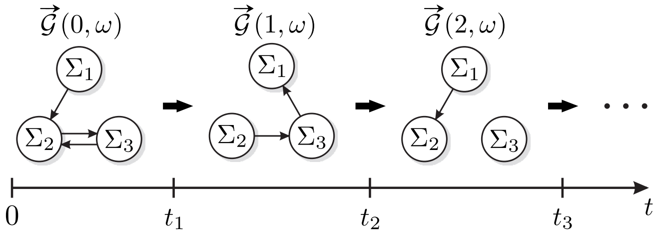

Figure 2: Random communication network modelled by a random sequence of graphs

with the stationary probability distribution

$$\text{Prob}\left(\vec{\mathcal{G}}(i, \omega) = \vec{\mathcal{G}}_j\right) = p_j, \qquad \sum_{j=1}^{q} p_j = 1$$

from a finite set of possible graphs $\Gamma = \{\vec{\mathcal{G}}_1, \dots, \vec{\mathcal{G}}_q\}$, where $\mathcal{L} = \{\boldsymbol{L}_1, \dots, \boldsymbol{L}_q\}$ denotes the set of associated Laplacian matrices. The switching times of the random communication network are either fixed with the switching of the network topology at $t_i = i \cdot T_s$ or random, i.e the switching times $t_i(\omega)$ are the result of a random experiment. The random communication network is described by the piecewise constant Laplacian matrix

$$\boldsymbol{L}(t, \omega) = \sum_{i=0}^{\infty} \boldsymbol{L}(i, \omega) \mathbf{1}_{[t_i, t_{i+1})}(t).$$

3 Multi-Agent System

The N agents of the multi-agent system are described by the continuous-time state-space model

$$\Sigma_i : \begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}\boldsymbol{x}_i(t) + \boldsymbol{b}u_i(t), & \boldsymbol{x}_i(0) = \boldsymbol{x}_{i0} \\ y_i(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}_i(t), & i = 1, \dots, N, \end{cases} \tag{1}$$

where $\boldsymbol{x}_i$ is the state vector, u_i the input signal and y_i the output signal of the i-th agent. The local control law of the i-th agent feeds back the difference of its output signal and the output signals of all neighbouring agents

$$C_i : \; u_i(t) = -k \sum_{j=1}^{N} l_{ij}(t, \omega) y_j(t), \quad i = 1, \dots, N. \tag{2}$$

The overall closed-loop system is obtained by aggregating the agents (1) and the local control laws (2):

$$\bar{\Sigma} : \begin{cases} \dot{\boldsymbol{x}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{A} - \boldsymbol{L}(t, \omega) \otimes k\boldsymbol{b}\boldsymbol{c}^{\mathrm{T}})\boldsymbol{x}(t) \\ \boldsymbol{y}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{c}^{\mathrm{T}})\boldsymbol{x}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0. \end{cases} \tag{3}$$

4 Project Aim

The aim of this project is to find new methods for the design of networked controllers of multi-agent systems with regard to random communication links. In order to solve this task the following two questions need to be answered:

1. **Asymptotic behaviour:** How can the synchronisation of the agents be ensured in case of random communication networks?

2. **Transient behaviour:** Can a specific choice of the available communication links improve the convergence rate?

Asymptotic behaviour. As the concept of pointwise convergence as employed for deterministic systems is generally not applicable for stochastic systems, the concept of almost sure convergence is used in which the convergence has to occur for almost all outcomes ω of the random experiment. Conditions on the design of the networked controller need to be derived that ensure the almost sure asymptotic synchronisation of the agents. Since the model of the closed-loop system (3) depends on the time and the outcome of an random experiment, a direct analysis of this system is complicated. Instead, the feedback gain k is designed so that the averaged closed-loop system

$$\bar{\Sigma}_{\text{ave}} : \begin{cases} \dot{\bar{\boldsymbol{x}}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{A} - \mathrm{E}(\boldsymbol{L}(t,\omega)) \otimes k\boldsymbol{b}\boldsymbol{c}^{\mathrm{T}})\bar{\boldsymbol{x}}(t) \\ \bar{\boldsymbol{y}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{c}^{\mathrm{T}})\bar{\boldsymbol{x}}(t), \quad \bar{\boldsymbol{x}}(0) = \boldsymbol{x}_0 \end{cases}$$

synchronises. However, to ensure the almost sure synchronisation of the agents, additional requirements are necessary. Either the switching time intervals $[t_i, t_{i+1})$ of communication network must be sufficiently small (see [2]) or the controlled agents must satisfy some form of monotonicity property (see [3]).

Transient behaviour. In order to improve the transient behaviour of the agents, an information measure $D_{i,\text{ref}}$ is introduced. The idea is that each agent sends not only its output signal but also the corresponding information measure to the neighbouring agents. Each local control unit then selects the best available output signal based on the information measure to control the agent. Hence, the emerging communication structure among the agents using only locally available information is the result of a self-organising process.

5 Experiments

Consider $N = 6$ mobile robots as they are employed at the experimental set-up SAMS whose movement is restricted to a circular path as shown in Fig. 3. The control objective is the platooning of the robots, i.e. the distances d_i between the robots need to be synchronised. The mobile robots are described by the model

$$\dot{\boldsymbol{x}}_i(t) = \begin{pmatrix} -0.5 & 0 & 0.005 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \boldsymbol{x}(t) + \begin{pmatrix} 0.1 \\ 0 \\ 1 \end{pmatrix} u_i(t)$$
$$y_i(t) = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \boldsymbol{x}_i(t), \quad \boldsymbol{x}_i(0) = \boldsymbol{x}_{i0}.$$

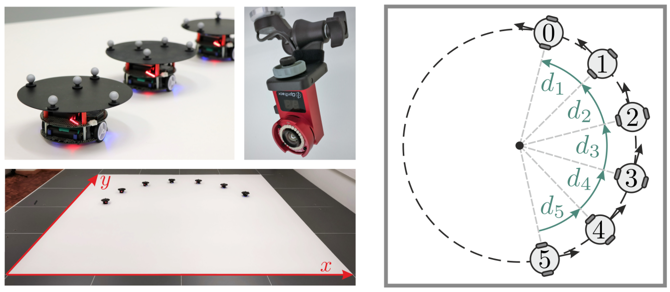

Figure 3: Experimental set-up SAMS (left) and synchronisation of the distances between mobile robots (right)

The communication network is modelled as a sequence of Erdös-Rényi graphs. Any communication link pointing from one agent to another agent exists with the fixed probability $p = 0.02$ in the network. The switching times $t_i = i \cdot T_{\mathrm{s}}$ of the random communication network are fixed with $T_{\mathrm{s}} = 0.01\,\mathrm{s}$. The feedback gain $k = 15$ of the local controllers (2) is designed in a way that the agents synchronise over the average communication network $\mathrm{E}(\boldsymbol{L}(t,\omega))$.

Since the random communication network fulfils the requirement of sufficiently fast switching, the distances between the robots synchronise, as shown in Fig. 4, which results in the desired platooning of the robots. In particular, the platooning emerges even though only a few communication links are active at each time step.

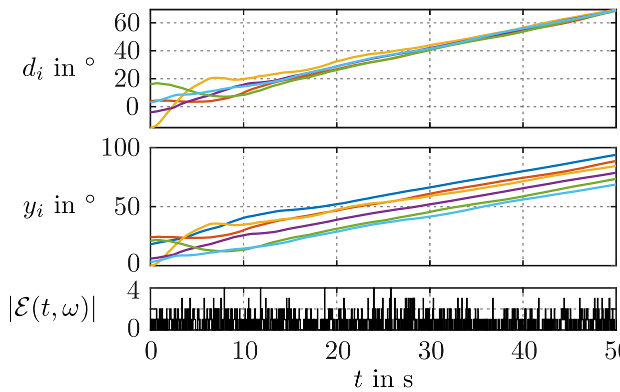

Figure 4: Distances (top) and position (middle) of the mobile robots and the number of active communication links over time (bottom)

References

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[2] P. Welz and J. Lunze. Synchronisation of linear agents over a time-varying random network. *6th International Conference on Control, Decision and Information Technologies*, Paris, 2019.

[3] P. Welz and J. Lunze. On synchronisation of internally positive agents over a random communication network. *Proc. of the Workshop on Distributed Estimation and Control in Networked Systems*. 2019. (submitted)