RUHR-UNIVERSITÄT BOCHUM
Institute of Automation and Computer Control
Prof. Dr.-Ing. Jan Lunze

Universitätsstr. 150 Phone +49-(0)234 32-28071
D-44805 Bochum Fax +49-(0)234 32-14101

Practical Synchronisation of Multi-Agent Systems

Marc Wissing
marc.wissing@iav.de

1 Introduction

Synchronisation is a common task in multi-agent systems. A networked controller should give all subsystems a common behaviour, which is governed by the agent dynamics. While the asympotic synchronisation of identical agents has been analysed extensively in the literature, the practical synchronisation of disturbed agents with model uncertainties has not been discussed. Especially, when thinking of the practical application of multi-agent systems, e.g. a vehicle platoon, a joint measurement task or the landing of a quadrotor on a moving object, disturbances and uncertainies can not be neglected and, hence, asymptotic synchronisation is difficult to achieve. Therefore, the main contribution of this project is to analyse the practical synchronisation and to find controllers and communication structures for this goal.

The project considers the mutual synchronisation of the agents P_i, $(i = 1, 2, ..., N)$, each of which can be affected by a disturbance $d_i(t)$, which is the output of a disturbance generator $\Sigma_{\mathrm{d}i}$ (Fig. 1). The networked controller consists of local controllers C_i and a communication network.

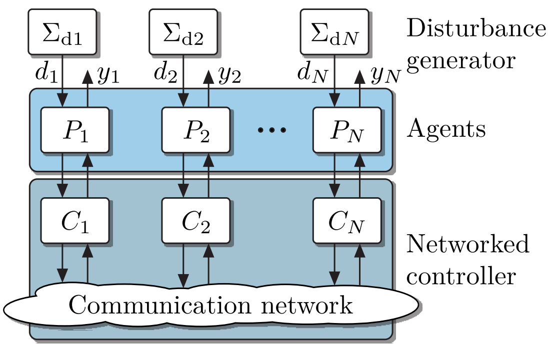

Figure 1: Networked control of multi-agent systems.

2 Project aim

The scientific goal of this project is to elaborate design methods for cooperative controllers that ensure practical synchronisation of linear agents with identical dynamics

$$\forall t \geq \bar{t}: \quad |y_i(t) - y_{\mathrm{s}}(t)| \leq \bar{e}, \quad i = 1, 2, ..., N. \tag{1}$$

In extension to the current state of the art, the control methods should be applicable to agents with bounded disturbances, with model uncertainties, and with control goals that include, besides synchronisation, additional practical requirements.

The main methodological problem consists in finding explicit relations between the performance requirements on the overall system behaviour and the properties of the controlled agents and the communication topology. The results should give design rules for the communication network to answer the question:

> Under what conditions should the controlled agents be connected by communication links?

As communication links require to transmit information with sufficient sampling frequency, precision and reliability, sparse communication graphs should be used.

The goals should be achieved under the restriction that the overall system has a shallow hierarchy without any coordinating unit. This fact is not only relevant for the online application of the control law, but also for the design phase in which the local controllers and the communication structure have to be selected without a complete model of the overall system, but with local information.

3 Synchronisation subject to disturbances

As a first step towards practical synchronisation, the asymptotic synchronisation of disturbed agents is considered, because almost no literature exists in this field. For arbitrary initial states $\boldsymbol{x}_{i0}$, $\boldsymbol{x}_{\mathrm{c}i0}$ and $\boldsymbol{x}_{\mathrm{d}i0}$ of the agents P_i, the local controllers C_i and the disturbance generators $\Sigma_{\mathrm{d}i}$, the agent outputs $y_i(t)$ should follow a synchronous trajectory $y_{\mathrm{s}}(t)$:

$$\lim_{t\to\infty} |y_i(t) - y_{\mathrm{s}}(t)| = 0, \quad i = 1, 2, ..., N.$$

This goal includes the synchronisation of undisturbed systems ($\boldsymbol{x}_{\mathrm{d}i0} = \boldsymbol{0}$), which is already discussed in [1], and the disturbance attenuation of initially synchronised systems ($\boldsymbol{x}_{i0} = \boldsymbol{x}_{j0},\ \forall i, j$) as particular cases [2,3]. Therefore, one main question of this project is:

> How to synchronise agents subject to disturbances?

In order to answer this question, results on disturbed multivariable control systems and networked control systems will be combined. Especially, an equivalent version of the internal-model principle in multivariable control for multi-agent systems is looked for. Known from multivariable control the open-loop system needs to include the distubance generator in order to attenuate disturbances at the output and, furthermore, the closed-loop system has to be asymptotically stable. Both conditions should be extended onto multi-agent systems. Therefore, the local controller is said to consist of two components (Fig. 2).

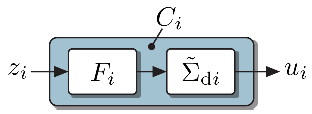

Figure 2: Local controller.

The first component $\tilde{\Sigma}_{\mathrm{d}i}$ brings in the missing dynamics of the disturbance generator

$$\tilde{\Sigma}_{\mathrm{d}i} = \Sigma_{\mathrm{d}i} \setminus \{P_i \cap \Sigma_{\mathrm{d}i}\}$$

into the open-loop system, which in multi-agent systems is the extended agent. The goal is to include the disturbance generator in the extended agent

$$\Sigma_{0i} \supseteq \Sigma_{\mathrm{d}i}.$$

In general the disturbance dynamics $\Sigma_{\mathrm{d}i}$ and with that the dynamics $\tilde{\Sigma}_{\mathrm{d}i}$ are not stable. Hence, adding $\Sigma_{\mathrm{d}i}$ into the extended agent makes it difficult to synchronise. Therefore, the second component F_i is introduced into the extended agent, which makes the extended agent output-feedback stabilisable and, hence, synchronises the overall system.

Having synchronising disturbed agents the question arises how the disturbances affects the synchronous trajectory $y_{\mathrm{s}}(t)$. Therefore, the synchronous behaviour subject to disturbances should be analysed and a relation between the disturbance generators and the synchronous trajectory is looked for.

Extension towards practical synchronisation. Towards practical synchronisation the disturbance generators cannot assumed to be exactly known. Therefore, model uncertainties for the disturbance generators have to be considerd. Expecially the relation between these model uncertainties and the synchronisation bound $\bar{e}$ in eqn. (1) is looked for. Further, local controllers C_i should be found that are robust to such uncertainties of the disturbance generators and, hence, lead to practical synchronisation.

4 Synchronisation of similar agents

Speaking of model uncertainties for the disturbance generators, it also cannot be assumed that the agents have identical dynamics. More likely the agents have the same structure with almost the same but different dynamics. Known from the literature on asymptotic synchronisation individual agents are only synchronisable if they have common dynamics. In the case of agents with similar dynamics as considered in this project none common dynamics exists and, hence, no asymptotic synchronisation is possible. Therefore the goal is to find a relation between the dynamical difference and the synchronisation bound $\bar{e}$.

5 Communication network

The communication network allows the local controllers to exchange information with each other. Therefore, a methodology to design the communication network is a main goal of this project. Known for the synchronisation of undisturbed agents with identical dynamics, it is necessary for the communication graph to possess a spanning tree. This leads to the question whether the same is the case for disturbed agents and similar agents or more conditions have to be satisfied by the communication graph.

6 Example: Oscillator network

Figure 3 shows the behaviour of a network of 5 identical oscillators. The example considers the synchronisation subject to disturbances, where the top figure shows the step disturbances for $t \geq 20\,\mathrm{s}$. The middle and bottom figure show the output behaviour of the oscillators for two different cases, where in the middle figure no internal model of the disturbances is included in the extended agent and in the bottom figure such internal model is included. In the first $20\,\mathrm{s}$ the oscillators synchronise in both cases. For $t \geq 20\,\mathrm{s}$ the extended agents without an internal model do not synchronise subject to disturbances, whereas for a satisfied internal-model principle the oscillators synchronise subject to disturbances.

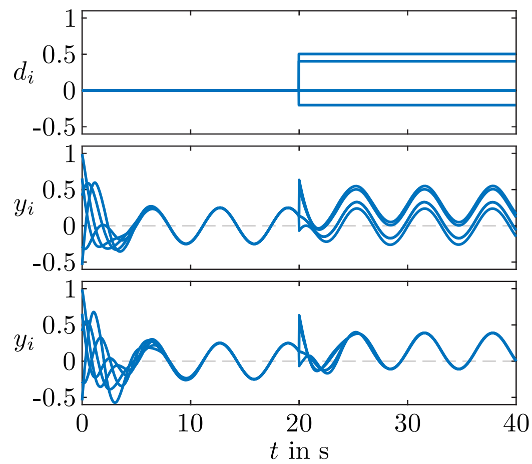

Figure 3: Step disturbances (top), oscillator network without local controller and oscillator network with networked controller which satisfies the internal-model principle.

The oscillator example can also be used for the analysis of the synchronisation of similar agents. Therefore, the oscillators have to be assumed to have different frequencies.

References

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