Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150
D-44805 Bochum, Germany

Phone +49 - (0)234 32 - 28071
Fax +49 - (0)234 32 - 14101

Controller design for periodically operated discretely controlled continuous systems

Dipl.-Ing. Axel Schild
schild@atp.rub.de

1 Discretely controlled systems

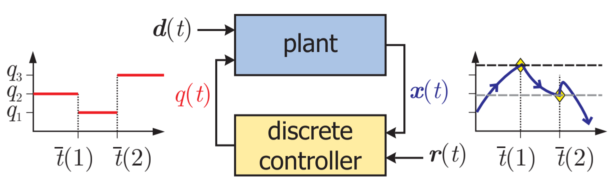


Figure 1: Structure of a discretely controlled continuous system

Discretely controlled continuous systems (DCCS) represent a relevant class of hybrid systems with many applications in process engineering, power electronics and robotics.
As shown in Fig. 1, such systems constitute a control loop composed of a continuous plant subject to disturbances $\boldsymbol{d}(t)$ and a discrete-event controller, which periodically switches the plant among its *modes of operation* $q \in Q$ to meet specifications imposed on the continuous state $\boldsymbol{x}(t)$ or the output $\boldsymbol{y}(t)$. For each mode q, the plant exhibits different continuous dynamics

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{d}(t), q(t)), \; \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{d}(t), q(t)) \quad . \tag{2}$$

The plant's modes are typically designed, such that at proper operation, the mode signal $q(t)$ periodically cycles through a finite sequence $Q_\Gamma = \left(\bar{q}_0^\star \ldots \bar{q}_{p-1}^\star\right)$.

2 Project goals

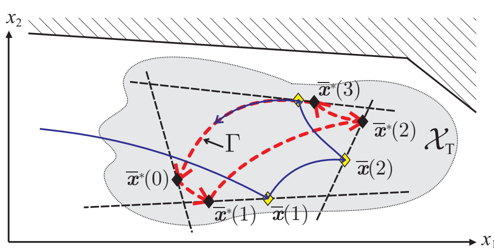


Figure 2: Stationary periodic operation of a DCCS

This project focusses on the stationary control of periodically operated DCCS (Fig. 2). **Given** the plant dynamics (1), (2), the control task is to **find** a controller that initially drives the plant's state $\boldsymbol{x}(t)$ into a specified *terminal region* $\mathcal{X}_T$ and there enforces asymptotic convergence towards a *predetermined p-periodic limit cycle* Γ. This must be achieved by solely adjusting a mode's activation duration from its nominal value $\bar{\tau}^\star\left(\bar{q}_k^\star\right)$.

The design procedure must explicitly account for effects of disturbances, large parameter variations and bounds on the switching frequency. Excessive degrees of design freedoms may be exploited to maximize the convergence rate,the robustness or to obey safety constraints on the continuous flow. Due to resource limitations, the control law must be a simple, explicit expression.

3 Control loop structures of DCCS

Two complementary approaches are considered here.

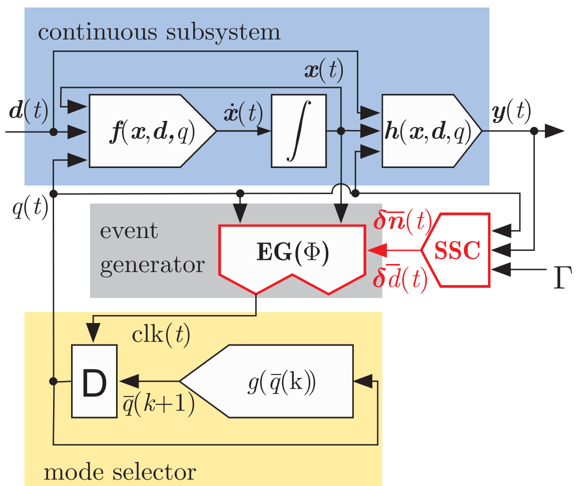


Figure 3: General control loop structure of DCCS

Stationary control by means of static switching surfaces. The discrete controller is structured into an *event generator* and a deterministic *switching logic* or *mode selector* (Fig. 3). The switching logic encodes the discrete transition structure, i.e. the set $Q'(q)$ of admissible successor modes to each $q \in Q$. Concerning periodically operated DCCS, $Q'(\bar{q}_k^\star) = \left\{\bar{q}_{k+1}^\star\right\}$ is a singleton and the mode selector degenerates into an autonomous cyclic automaton.
The event generator constantly evaluates an event function $\Phi(\boldsymbol{x}, q)$, which encodes information about *when* to switch. Upon satisfaction of $\Phi(\boldsymbol{x}, q) = 0$, an impulse $\mathrm{clk}(\bar{t}(k)) = 1$ is triggered at the *switching instant* $\bar{t}(k)$, causing a mode transition. The set

$$\mathcal{S}(q) = \{\boldsymbol{x} \mid \Phi(\boldsymbol{x}, q) = 0\}$$

of all states $\bar{\boldsymbol{x}}$ satisfying the switching condition of mode q, constitutes a *static switching surfaces* in state space. The main design issue is to find static surfaces $\mathcal{S}(q)$, which guarantees desired loop properties.

Stationary control by means of dynamic switching surface adjustment. The primary control loop can be augmented by an additional outer control loop, whose central component is a *switching surface controller* (SSC, Fig. 3), which acts on the event generator at runtime. The SSC dynamically adjusts the event function parameters, thereby affecting the orientation and location of nominal switching surfaces $\mathcal{S}_0(q)$ (Fig. 4). The central idea is to pick simple nominal switching planes

$$\mathcal{S}_0(q) = \left\{ \boldsymbol{x} \mid \boldsymbol{n}_0^{\mathrm{T}}(q)\,\boldsymbol{x} - d_0(q) = 0 \right\}$$

and exploit the supplementary control input for the dynamic local reconstruction of more complex switching surfaces by adjusting the orientations $\boldsymbol{n}_0^{\mathrm{T}}(q)$ as well as the positions $d_0(q)$. Here, the design concentrates on the model-based derivation of the switching surface controller.

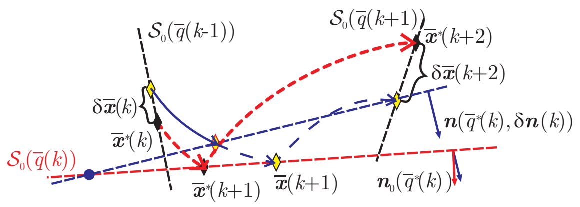


Figure 4: On-line adjustment of switching surfaces

4 Design of static switching planes

The key element for obtaining a model that explicitly reflects the effect of the design parameters $\boldsymbol{n}^{\mathrm{T}}(q)$ and $d(q)$ on the evolution of $\boldsymbol{x}(t)$, is to sample the system behavior at switching instants $\bar{t}(k)$. In theory, this sampled description is achieved by the embedded map [1]

$$\begin{pmatrix}\bar{\boldsymbol{x}}(k+1) & \bar{q}(k+1) & \bar{\tau}(k+1)\end{pmatrix}^{\mathrm{T}} = \boldsymbol{H}\left(\begin{pmatrix}\bar{\boldsymbol{x}}(k) & \bar{q}(k) & \bar{\tau}(k)\end{pmatrix}^{\mathrm{T}}\right) \quad (3)$$

with $\bar{\boldsymbol{x}}(k), \bar{q}(k), \bar{\tau}(k)$ denoting the switch point, the activated mode and the activation duration of this mode at the k-th switching. Although a closed form expression of (3) typically does not exist, the map's continuous part $\boldsymbol{H}_x$ can still be expanded as a first-order Taylor series

$$\begin{aligned}\delta\bar{\boldsymbol{x}}(k+1) &= \tfrac{\mathrm{d}\boldsymbol{H}_x}{\mathrm{d}\boldsymbol{x}}(\bar{\boldsymbol{x}}^\star(k), \bar{q}(k), \bar{\tau}^\star(k))\,\delta\bar{\boldsymbol{x}}(k)\\ &\left(\boldsymbol{I} - \frac{\boldsymbol{f}(\bar{\boldsymbol{x}}^\star(k+1), \bar{q}(k))\,\boldsymbol{n}^{\mathrm{T}}(\bar{q}(k))}{\boldsymbol{n}^{\mathrm{T}}(\bar{q}(k))\,\boldsymbol{f}(\bar{\boldsymbol{x}}^\star(k+1), \bar{q}(k))}\right)\frac{\mathrm{d}\bar{\boldsymbol{x}}^\star(k+1)}{\mathrm{d}\bar{\boldsymbol{x}}^\star(k)}\delta\bar{\boldsymbol{x}}(k)\,,\end{aligned}$$

with $\delta\bar{\boldsymbol{x}}(k) = \bar{\boldsymbol{x}}(k) - \bar{\boldsymbol{x}}^\star(k)$ denoting the deviation from the limit cycle. An iterated application of the embedded map over a complete cycle Q_Γ yields a linear approximation

$$\delta\bar{\boldsymbol{x}}(c+1) = \tfrac{\mathrm{d}\boldsymbol{P}_x}{\mathrm{d}\boldsymbol{x}}(\bar{\boldsymbol{x}}^\star(\bar{q}(c)), \bar{q}(c), \bar{\tau}^\star(\bar{q}(c)))\delta\bar{\boldsymbol{x}}(c) \quad , \qquad (4)$$

of a return map $\boldsymbol{P}$. As shown in [3], map (4) can be interpreted in terms of a discrete-time periodic linear system

$$\boldsymbol{\zeta}(k+1) = \boldsymbol{A}(k)\boldsymbol{\zeta}(k) + \boldsymbol{b}(k)u(k) \qquad (5)$$

$$u(k) = -\boldsymbol{k}^{\mathrm{T}}(k)\boldsymbol{\zeta}(k) \qquad (6)$$

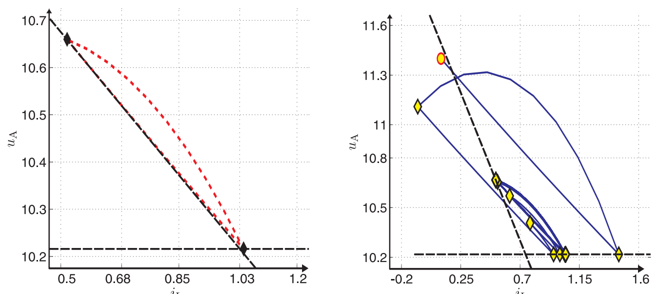
with static state feedback, where $\boldsymbol{k}^{\mathrm{T}}(k)$ is uniquely related to the normal $\boldsymbol{n}^{\mathrm{T}}(\bar{q}_k^\star)$, iff the constraint $\boldsymbol{k}^{\mathrm{T}}(k)\boldsymbol{f}\left(\bar{\boldsymbol{x}}^\star(\bar{q}_{k+1}^\star), \bar{q}_k^\star\right) = 1$ is satisfied. This equivalence allows to conclude about local orbital stability of Γ from the eigenvalues of the monodromy matrix of (5) as well as to apply well developed methods from periodic control systems for solving the original problem [3].

5 Switching surface controller design

Given nominal switching planes, another locally valid linear periodic model can be derived that enables the model-based design of a switching surface controller [2] for stabilizing the limit cycle Γ. Since using a linear approximation of (3) in the design, both control strategies result in a similar loop performance, i.e. they complement each other. However, by combining the static plane design with a different synthesis approach for the SSC offers a strong potential for significantly enlarging Γ's region of attraction. To compensate for varying parameters, an on-line adjustment of the event function parameters is indispensable anyway.

6 Application example

Both design approaches can be applied to stabilize the periodic operation of a DC-DC boost converter in continuous conduction mode. The static switching surface design results in the switching planes depicted in Figure 5(a). Alternatively, paraxial nominal switching planes $\mathcal{S}_0(q)$ can be adjusted on-line by a SSC to stabilize the otherwise marginally stable limit cycle. A sample execution for this control approach is depicted in Figure 5(b), where the point $\bar{\boldsymbol{x}}(k)$ to the left are off the nominal plane $\mathcal{S}_0(1)$ due to the interference of the SSC.



(a) static switching surfaces

(b) switching sequence under application of SSC

Figure 5: Stabilized limit cycle of a DC-DC Boost converter

References

[1] J. Krupar, J. Lunze, W. Schwarz, and A. Schild. Modelling and analysis of discretely controlled continuous systems by means of embedded maps. *IEICE Transactions on Fundamentals of Electronics, Communication and Computer Science*, 89:2697–2705, 2006.

[2] A. Schild and J. Lunze. Stabilization of limit cycles of discretely controlled continuous systems by controlling switching surfaces. In *Hybrid Systems: Computation and Control (HSCC 2007)*, volume 4416 of *LNCS*, pages 515–528, 2007.

[3] A. Schild and J. Lunze. Switching surface design for periodically operated discretely controlled continuous systems. In *Hybrid System: Computation and Control*, pages 471–485. Springer Verlag, Heidelberg, 2008.