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A comprehensive observer-based fault isolation procedure

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1 Introduction

Mechatronic systems exhibit an ever increasing level of automation and complexity. A malfunction of such systems may cause significant damage, pollution or danger to humans. Supervision is mandatory in order to maintain safety and reliability, and to fulfill legal requirements. The task of diagnostic algorithms is to detect and isolate faults that may affect the system. Fault detection is concerned with verifying whether or not a system is faulty. Fault isolation tries to find the faulty component in the system.

Mechatronic systems consist of various components, including actuators, sensors, and control units. Due to the size of those systems it is no longer advisable to design diagnostic functions that are based on heuristic analysis or expert knowledge. Instead, systematic methods have to be developed in order

- to analyze whether or not faults are diagnosable, i.e., detectable or isolable, respectively, and
- to build diagnostic algorithms.

The project aim is to provide a comprehensive fault isolation procedure which tackles the two problems.

2 Model-based fault diagnosis

Fault diagnosis of a system Σ is possible only if the knowledge about the system includes redundancies. In model-based diagnosis, the mathematical model together with the known input and output signals of the system provides redundant information (Fig. 1). A first step towards a diagnosis algorithm is to perform a diagnosability analysis, i.e. to analyze which part of the system contains redundancies.

Structural analysis has become popular in model-based diagnosis to analyze diagnosability [1, 3]. It is based on graph theory and identifies over-determined subsets of equations within the system model which include redundancies and make fault diagnosis possible.

Diagnostic algorithms use the known input and output signals of a system to compute residuals (signal $\boldsymbol{r}(t)$ in Fig. 1) that indicate whether the system behavior is consistent with the model of the non-faulty system. By evaluating these residuals appropriately, the fault isolation task can be accomplished.

Most literature deals with particular aspects of fault diagnosis, i.e., either diagnosability analysis or residual generation or residual evaluation. The aim of the present

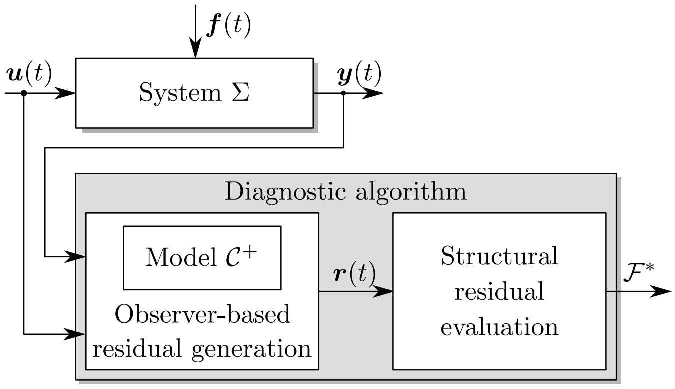

Figure 1: Diagnostic algorithm

project is to provide a concept that leads through the entire process of building a fault diagnoser for a given mechatronic system. The starting point is a mathematical description of the system by means of a state space model. The result shall be a diagnoser that is able to detect and isolate faults of a predefined fault set $\mathcal{F}$.

3 A comprehensive fault isolation procedure

3.1 Structural diagnosability analysis and residual evaluation logic

It is assumed that the system can be described by a linear state space model

$$\Sigma : \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}_{\mathrm{s}}\boldsymbol{f}(t), & \boldsymbol{x}(0) = \boldsymbol{x}_0, \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{E}_{\mathrm{o}}\boldsymbol{f}(t), \end{cases}$$

with state $\boldsymbol{x}(t) \in \mathbb{R}^n$, input $\boldsymbol{u}(t) \in \mathbb{R}^m$, measurement $\boldsymbol{y}(t) \in \mathbb{R}^p$, and a fault vector $\boldsymbol{f}(t) \in \mathbb{R}^l$. The evolution of $\boldsymbol{f}(t)$ is unknown, but the matrices $\boldsymbol{E}_{\mathrm{s}}$ and $\boldsymbol{E}_{\mathrm{o}}$ are supposed to be known and describe how the faults $f_1(t), \ldots, f_l(t)$ affect the system.

The first task is to analyze diagnosability of the system, i.e., to identify subsets of equations within Σ which contain more equations than unknown variables. For this, equations and variables are subsumed in the *constraint set* $\mathcal{C}$ and the *variable set* $\mathcal{Z}$, respectively. A bipartite graph shows the relations between the elements of $\mathcal{C}$ and $\mathcal{Z}$, see Fig. 2. In such a graph, equations are represented as bars and variables as circles. White, gray, and red circles correspond to unknown, known, and fault variables,

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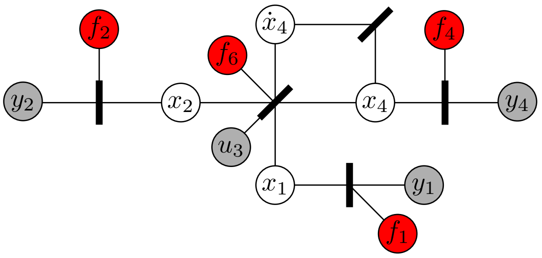

Figure 2: Example of a bipartite structure graph

respectively. An edge is drawn from an equation vertex to a variable vertex if the variable is contained in the equation. With a DULMAGE-MENDELSOHN decomposition, the unique maximal subset $\mathcal{C}^+ \subseteq \mathcal{C}$ can be computed which has the property that it is over-determined. Within $\mathcal{C}^+$, further over-determined subsets $P_\kappa \subseteq \mathcal{C}^+$, $\kappa = 1, \dots, k$, can be computed [2]. All over-determined subsets are subsumed in the set $\mathcal{P} \subseteq 2^{\mathcal{C}}$.

The dependencies of sets P on faults f are summarized in the Boolean *fault signature matrix* $\boldsymbol{S} = (s_{\kappa j})$. If f_j is contained in an equation of P_κ, then $s_{\kappa j}$ is 1, else it is 0. An example of a fault signature matrix is given in Fig. 3.

The *signature* of a fault f_j is given by the jth column $\boldsymbol{s}_j$ of the matrix. A fault f_j is called *structurally detectable* if $\boldsymbol{s}_j \neq \boldsymbol{0}$, and *structurally isolable* if $\boldsymbol{s}_i \neq \boldsymbol{s}_j$ for all $i \neq j$.

$\boldsymbol{S} =$

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
P_1	1	0	0	1	1	1	1	1	0
P_2	0	1	1	0	1	1	1	1	0
P_3	1	1	1	0	1	0	0	0	0
P_4	1	1	0	1	0	1	0	0	0

Figure 3: Example of a fault signature matrix

It is assumed that from a selection $\mathcal{Q} \subset \mathcal{P}$, $|\mathcal{Q}| = q$, of over-determined subsets, residuals can be computed. Each residual $\boldsymbol{r}_\kappa(t)$, $\kappa = 1, \dots, q$, is expected to react on faults according to the fault signature matrix: If $s_{\kappa j} = 1$, then $\boldsymbol{r}_\kappa(t) \neq 0$ in case that $f_j(t) \neq 0$. In all other situations, $\boldsymbol{r}_\kappa(t)$ will converge to zero.

For each $\boldsymbol{r}_\kappa(\cdot)$, a norm $\|\cdot\|_\kappa$ and a threshold $\varepsilon_\kappa > 0$ has to be chosen, and a Boolean vector $\bar{\boldsymbol{r}}(t) = (\bar{r}_1(t), \dots, \bar{r}_k(t))^{\mathrm{T}}$ is defined as follows:

$$\begin{aligned} \bar{r}_\kappa(t) &= 0, \quad \text{if} \quad \|\boldsymbol{r}_\kappa(t)\|_\kappa \leq \varepsilon_\kappa, \\ \bar{r}_\kappa(t) &= 1, \quad \text{if} \quad \|\boldsymbol{r}_\kappa(t)\|_\kappa > \varepsilon_\kappa. \end{aligned}$$

The vector $\bar{\boldsymbol{r}}(t)$ is compared with the columns $\boldsymbol{s}_j$ of the fault signature matrix online and each column which matches the residual vector provides a fault candidate. Thus, a set of fault candidates

$$\mathcal{F}^*(t) := \{f_j \mid \boldsymbol{s}_j = \bar{\boldsymbol{r}}(t)\}$$

is determined by the fault diagnoser.

3.2 Observer-based residual generation

Once the selection $\mathcal{Q} \subset \mathcal{P}$ is made, a residual has to be computed from each $P_\kappa \in \mathcal{Q}$. This shall be done by state estimation. Therefore, a state space model

$$\Sigma_\kappa : \begin{cases} \dot{\boldsymbol{x}}_\kappa(t) = \boldsymbol{A}_\kappa \boldsymbol{x}_\kappa(t) + \boldsymbol{B}_\kappa \boldsymbol{u}_\kappa(t), \quad \boldsymbol{x}_\kappa(0) = \boldsymbol{x}_{\kappa,0}, \\ \boldsymbol{y}_\kappa(t) = \boldsymbol{C}_\kappa \boldsymbol{x}_\kappa(t) \end{cases}$$

has to be set up from the equations of P_κ for each $\kappa = 1, \dots, q$. An observer has to be constructed for this subsystem which is of the form

$$O_\kappa : \begin{cases} \dot{\hat{\boldsymbol{x}}}_\kappa(t) = \boldsymbol{A}_\kappa \hat{\boldsymbol{x}}_\kappa(t) + \boldsymbol{B}_\kappa \boldsymbol{u}_\kappa(t) + \boldsymbol{L}_\kappa \boldsymbol{r}_\kappa(t), \\ \boldsymbol{r}_\kappa(t) = \boldsymbol{y}_\kappa(t) - \boldsymbol{C}_\kappa \hat{\boldsymbol{x}}_\kappa(t). \end{cases}$$

with a feedback matrix $\boldsymbol{L}$ which has to be chosen such that O_κ is an asymptotically stable system. The observer delivers the residual $\boldsymbol{r}_\kappa(t)$ which is used for fault diagnosis.

3.3 Systematic construction of a fault diagnoser

Given a state space model of a mechatronic system, the comprehensive diagnosis approach can be summarized as follows:

1. Set-up of a structural model (cf. Fig. 2)
2. Computation of all over-determined subsets of $\mathcal{C}^+$
3. Computation of the fault signature matrix $\boldsymbol{S}$
4. Selection of a subset $\mathcal{Q}$ of over-determined sets
5. Set-up of observer-based residual generators for all $P \in \mathcal{Q}$

Result. The result is a bank of observer-based residual generators which are designed to estimate the state of the respective subsystem Σ_κ and to provide a residual. The residual vector $\bar{\boldsymbol{r}}(t)$ is evaluated on-line and compared with the fault signature matrix $\boldsymbol{S}_{\mathcal{Q}}$ in order to provide a set $\mathcal{F}^*$ of fault candidates that may have occurred.

4 Discussion

There are several critical points in the above procedure.

1. How to choose the subset $\mathcal{Q}$ within $\mathcal{P}$?
2. How to verify whether a set $P \in \mathcal{P}$ defines a state space model?
3. How to assure that a residual shows the behavior as predicted by the fault signature matrix?
4. How to deal with nonlinear and hybrid systems?

Future research will cover these topics.

References

[1] Blanke, M., Kinnaert, M., Lunze, J., and Staroswiecki, M. (2016). *Diagnosis and Fault-Tolerant Control.* Springer, Heidelberg.

[2] Krysander, M., Aslund, J., and Nyberg, M. (2008). An efficient algorithm for finding minimal over-constrained subsystems for model-based diagnosis. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 38(1), pp. 197–206.

[3] Pröll, S., Jarmolowitz, F., Lunze, J., Structural diagnosability analysis of switched systems, *9th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, Paris 2015, pp. 156–163