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Discrete-event control design for plants modeled by I/O automata

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1 Project aims

The fact that technological systems work under an actio-reactio principle motivates this project. A plant is modeled in this framework as an I/O automaton $\mathcal{N}_p$ which receives control inputs $v_p = w_c$, generates the output w_p and moves from state z_p to z'_p (Fig. 1). The main objective is to design the controller $\mathcal{A}_c$ which steers the plant $\mathcal{N}_p$ according to the sensor measurement v_c and the specification $\mathcal{S}$ e.g. to reach a final state z_F.

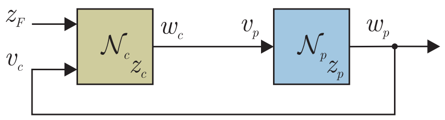

Figure 1: Control loop of I/O automata

The goal of this work is to develop an I/O control design framework. The events v_p are assumed to be controllable whereas the events w_c, w_p and v_c are assumed to be observable. Unobservable and uncontrollable events do not contribute with any useful information for the control design, hence there is no need to model them explicitly here. To set up this new theory, the following issues have to be formalized in an I/O automata framework:

1. Feasibility of the specification,
2. Controllability of the specification,
3. Well-posedness of possibly emerging algebraic loops,
4. Uniqueness of control policy and
5. Nonblockingness of the control loop.

Investigations are necessary to clarify how these properties are related to each other. In the first project phase, a control design method is developed for a controller that only receives the sensor values v_c. The extension will consists of an additional specification input z_F to the controller as shown in Fig. 1.

2 Formal framework

The plant model consists of a nondeterministic I/O automaton $\mathcal{N}_p = (\mathcal{Z}_p, \mathcal{V}_p, \mathcal{W}_p, L_p, \mathcal{Z}_{0p})$ where $\mathcal{Z}_p, \mathcal{V}_p$ and $\mathcal{W}_p$ respectively represent the set of states, the set of inputs and the set of outputs. L_p is the characteristic function of the behavioral relation $\mathcal{L}_p$, thus exhibits the dynamics of the system.

$$L_p : \mathcal{Z}_p \times \mathcal{W}_p \times \mathcal{Z}_p \times \mathcal{V}_p \to \{0, 1\}$$

$$L_p(z'_p, w_p, z_p, v_p) = \begin{cases} 1, & \text{if } (z'_p, w_p, z_p, v_p) \in \mathcal{L}_p \\ 0, & \text{else.} \end{cases}$$

$\mathcal{Z}_{0p}$ is the set of initial states. The $*$ symbol in the argument of $L(\cdot)$ means that any value of the corresponding element can be considered according to its symbol set. According to the context, the subscript x of $\mathcal{N}_x$ will be replaced by a c for the controller, an s for the specification or a p for the plant.
In order to address specific elements of an I/O automaton the sets

$$\mathcal{V}_{ax}(z'_x, z_x) = \{v_x \in \mathcal{V}_x : L_x(z'_x, *, z_x, v_x) = 1\} \quad (1)$$

$$\mathcal{W}_{ax}(z'_x, z_x) = \{w_x \in \mathcal{W}_x | \exists v_x \in \mathcal{V}_x : L_x(z'_x, w_x, z_x, v_x) = 1\} \quad (2)$$

consists respectively of *active inputs* or *active outputs* of $\mathcal{N}_x$ from a state z_x or a state couple (z'_x, z_x).

Well-posedness. A control is said to be well-posed if neither the plant nor the controller can block for a given input combination (v_c, v_p) from the states (z_c, z_p). Since both automata $\mathcal{N}_p$ and $\mathcal{N}_c$ must switch to their respective next states (z'_c, z'_p), an algebraic loop emerges because of the feedback connection. Possible solutions of the algebraic loop have been first discussed in [1], then applied in [2] for control loop analysis.

3 Specification modeling

The specification automaton $\mathcal{A}_s$ is obtained by applying the specification operator Spec$(\cdot)$ on the automaton plant $\mathcal{N}_p$ w.r.t. $\mathcal{S}$, that is

$$\mathcal{A}_s = \text{Spec}(\mathcal{N}_p, \mathcal{S}). \quad (3)$$

The specification $\mathcal{S}$ is expressed as follows:

- a specific final state z_F to reach.
- a state sequence $Z_s(0 \cdots k_e)$ to follow.
- an output sequence $W_s(0 \cdots k_e)$ to generate.
- a set of illegal states $\mathcal{Z}_{ill}$ to avoid.
- a set of illegal outputs $\mathcal{W}_{ill}$ not to generate.
- a set of illegal I/O transitions $\mathcal{T}_{ill}$ to avoid.

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Moreover a specification can be given in form of periodical state or output sequence such as $Z^*_{spec}(0 \cdots k_e)$ or $W^*_{spec}(0 \cdots k_e)$. In this case, the corresponding control law, if it exists, just need to be applied periodically. Note that the specification of Type z_F is equivalent to the marked state concept widely used in automata theory, which shows that this specification modeling method used here also hold for other automata category.

4 I/O controller design

The design of the I/O controller automaton $\mathcal{N}_c = (\mathcal{Z}_c, \mathcal{V}_c, \mathcal{W}_c, L_c, \mathcal{Z}_{0c})$ consists of the following steps [2]:

1. Build the specification automaton $\mathcal{A}_s$ (3).

2. According to the signal flow of Fig. 1, define the sets

$$\mathcal{Z}_c = \mathcal{Z}_s, \mathcal{V}_c = \mathcal{W}_s, \text{ and } \mathcal{W}_c = \mathcal{V}_s. \tag{4}$$

3. Build the characteristic function L_c with

$$L_c(z'_c, w_c, z_c, v_c) = \begin{cases} 1, & \text{if } L_s(z'_c, v_c, z_c, w_c) \in \mathcal{L}_s \\ 0, & \text{else.} \end{cases} \tag{5}$$

Since $\mathcal{N}_c$ may become nondeterministic even if it enforces $\mathcal{S}$, deterministic control laws $\mathcal{A}_c^{(i)}$ with $i = 1, \ldots, \nu$ are derived from $\mathcal{N}_c$. They also enforce $\mathcal{S}$ and maintain the well-posedness of the control loop. Moreover an explicit realization scheme depicted in Fig. 2 has been developed in [2] to reflect the behavior of the control loop. The internal counter k_s of the controller is incremented only if the received measurement of the plant is in adequation with the expected ones in the controller. According the value of the internal counter, the current state $Z_s(k)$ and the next states $Z_s(k+1)$ to be achieved by the plant are updated. The active output and active input signals of the current state couple $(Z_s(k+1), Z_s(k))$ are evaluated w.r.t. (1) and (2) to determine the control signal $w_c(k)$ and the expected inputs $\hat{\mathcal{V}}_c(k)$. In case of unexpected sensor values from the plant, the internal counter k_s is not incremented and the control output $w_c(k)$ remains unchanged until a correct input is registered.

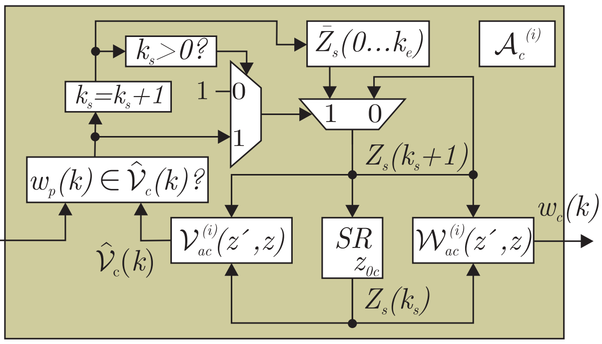
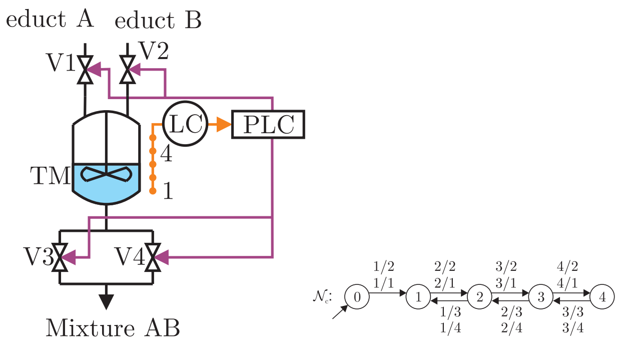


Figure 2: Realization of the feedback controller

5 Example: Cyclic level control

The objective of the process is to fill the tank TM from level 0 up to level 4, then down to level 1 and back to level 4 in a cyclic way. The symbol $w_c = v_p = 0$ models the `close-all-valves` command whereas $w_c = v_p = i$ is the `open-valve-Vi` command. The states z_p of the plant $\mathcal{N}_p$ are modeled by the states of the tank TM which varies from 0 (empty) up to 5 (full). Five sensors permit a discrete measurement of the level of the tank TM at discrete time steps modeled by $v_c = w_p$. w_p represents the expected result of an inflow or an outflow of the educt.



(a) Chemical plant (b) I/O controller automaton

Figure 3: Controlled chemical plant

The nominal constraint is formalized in the following by a specification $\mathcal{S}$ of type Z_s in the form $\mathcal{S} : Z_s(0 \cdots k_e) = (0, (1, 2, 3, 4, 3, 2)^*)$. The control design method introduced above leads to the controller automaton $\mathcal{A}_c$ w.r.t. (4) and (5) (Fig. 3(b)). Experimental results of the nominal behavior can be visualized in Fig. 4.

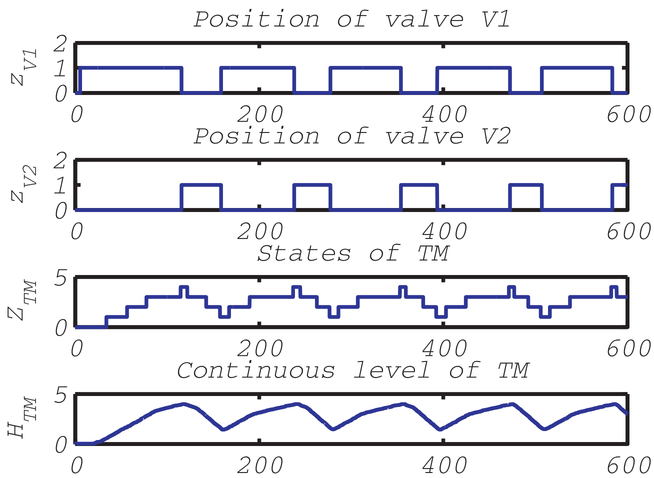


Figure 4: Experimental level control of Tank TM

References

[1] Y. Nke, S. Drüppel, and J. Lunze. Direct feedback in asynchronous networks of input-output automata. In *Proc. 10th European Control Conference*, Budapest, Hungary, 2009.

[2] Y. Nke and J. Lunze. Control design for nondeterministic input/output automata. In *Proceedings of the 18th IFAC Congress*, Milan, Italy, 2011.