Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150
D-44805 Bochum, Germany

Phone +49 - (0)234 32 - 28071
Fax +49 - (0)234 32 - 14101

Markov-Parameter-Based Input/Output-Reconfiguration

Dipl.-Ing. Jan Richter
richter@atp.rub.de

1 Introduction

The reconfiguration of automatic controllers in response to faulty plants is a central topic within the fault-tolerant control framework [1, 2]. The task of control reconfiguration is to automatically find a new controller after the occurrence of a fault, such that the reconfigured closed loop approximately satisfies the original set of control specifications.
In this project, the control reconfiguration of linear plants

$$\Sigma_P : \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{B}_{\boldsymbol{d}}\boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}_c(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases} \tag{1}$$

after actuator and sensor faults expressed by modified input and output maps $\boldsymbol{B}_f$ and $\boldsymbol{B}_f$ was investigated. A static reconfiguration block was considered first (Figure 1), followed by a design method for its generalisation, the dynamic virtual actuator [3–5].
The objective consists in reaching the trajectory recovery goal, which requires that the reconfigured closed loop has the same transmission dynamics from reference to output as the nominal loop.

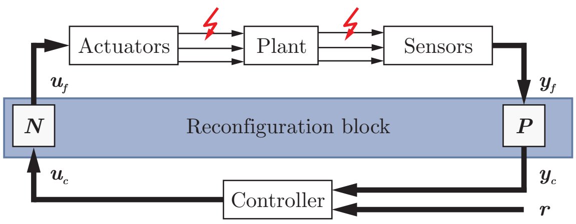

Figure 1: Statically reconfigured closed-loop system.

2 Input/output reconfiguration after actuator faults

The notion of static I/O-reconfigurability is defined as follows.

Definition 1 (Static I/O-reconfigurability after actuator faults)**.** A plant is *statically I/O-reconfigurable* after actuator or sensor faults, if there exist matrices $\boldsymbol{N}$ and $\boldsymbol{P}$ such that the static reconfiguration $\boldsymbol{u}_f(t) = \boldsymbol{N}\boldsymbol{u}_c(t)$, $\boldsymbol{y}_c(t) = \boldsymbol{P}\boldsymbol{y}_f(t)$ completely recovers its I/O-behaviour

$$\forall t > 0, \boldsymbol{u}_c(t), \boldsymbol{d}(t) : \boldsymbol{y}(t) - \boldsymbol{y}_f(t) = \boldsymbol{0},$$

where $\boldsymbol{y}$ and $\boldsymbol{y}_f$ denote the nominal response to an input $\boldsymbol{u}_c$ or disturbance $\boldsymbol{d}$ and the response of the faulty plant to the same input. □

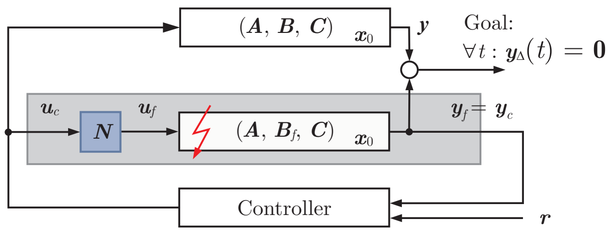

Figure 2: Static I/O reconfiguration goal for actuator faults.

The idea is illustrated for actuator faults in Figure 2. The static block is to be designed such that the difference between the outputs of the (fictitious) nominal plant and reconfigured plant vanishes.
The solution is formulated in terms of the Markov parameters $\boldsymbol{C}\boldsymbol{A}^i\boldsymbol{B}, \quad i = 0, \ldots, n-1$ of the nominal and faulty plants. They represent a complete description of a system's I/O-behaviour and are hence well suitable for its recovery. The solvability condition and the solution are formulated in the following Theorem.

Theorem 1 (Static I/O-reconfiguration after actuator faults)**.** *A plant is statically I/O-reconfigurable after actuator faults, if and only if it fulfils the rank condition*

$$\text{rank}\left(\boldsymbol{S}_{\text{O}}\boldsymbol{B}_f\right) = \text{rank}\left(\boldsymbol{S}_{\text{O}}\boldsymbol{B}_f \quad \boldsymbol{S}_{\text{O}}\boldsymbol{B}\right) \tag{2}$$

with the observability matrix $\boldsymbol{S}_{\text{O}}$*. The solution is*

$$\boldsymbol{u}_f(t) = \boldsymbol{N}\boldsymbol{u}_c(t), \quad \boldsymbol{N} = (\boldsymbol{S}_{\text{O}}\boldsymbol{B}_f)^+\boldsymbol{S}_{\text{O}}\boldsymbol{B}. \tag{3}$$

In [4] it is shown that the reconfigured loop is internally stable, if the nominal loop was internally stable and the faulty plant is stabilisable, and the sensitivity of the approach to numerical rank deficiencies is studied. It is found that rank test failures due to numerical reasons still permit the successful use of this approach.

3 Input/output reconfiguration after sensor faults

The reconfiguration problem after sensor faults is dual to the problem after actuator faults. Dual results are obtained, which however differentiate with respect to inputs and disturbances.

Theorem 2 (Static trajectory recovery after sensor faults)**.** *The reconfiguration problem after sensor faults is solvable with respect to the reference behaviour if and only if*

$$\text{rank}\left(\boldsymbol{C}_f\boldsymbol{S}_{\text{C}}\right) = \text{rank}\begin{pmatrix} \boldsymbol{C}_f\boldsymbol{S}_{\text{C}} \\ \boldsymbol{C}\boldsymbol{S}_{\text{C}} \end{pmatrix}$$

holds and is solved by the static block

$$\boldsymbol{P} = \boldsymbol{C}\boldsymbol{S}_{\mathrm{C}}(\boldsymbol{C}_f\boldsymbol{S}_{\mathrm{C}})^+,$$

where $\boldsymbol{S}_{\mathrm{C}}$ *is the controllability matrix with respect to the input* $\boldsymbol{u}_c$*. The problem is solvable with respect to the disturbance behaviour if and only if*

$$\operatorname{rank}\left(\boldsymbol{C}_f\boldsymbol{S}_{\mathrm{C,d}}\right) = \operatorname{rank}\begin{pmatrix}\boldsymbol{C}_f\boldsymbol{S}_{\mathrm{C,d}}\\ \boldsymbol{C}\boldsymbol{S}_{\mathrm{C,d}}\end{pmatrix}$$

holds and solved by the static block

$$\boldsymbol{P_d} = \boldsymbol{C}\boldsymbol{S}_{\mathrm{C,d}}(\boldsymbol{C}_f\boldsymbol{S}_{\mathrm{C,d}})^+, \tag{4}$$

where $\boldsymbol{S}_{\mathrm{C,d}}$ *is the controllability matrix with respect to the disturbance input* $\boldsymbol{d}$*.*

4 Example: Two-tank system

The plant consists of two tanks T1, T2 interconnected by valves with the control inputs u_L and u_U, where T1 is filled via pump u_P as shown in Figure 3. Valves are electromechanically driven with the motor states v_L and v_U. The controlled quantities are the levels h_1 and h_2. The state is $\boldsymbol{x} = \begin{pmatrix} v_L & v_U & h_1 & h_2\end{pmatrix}^T$. The tank system is described by the linear model (1) with

$$\boldsymbol{A} = 10^3\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -3.2 & -3.4 & -7.1 & 3.6 & 0 & 0\\ 3.2 & 3.4 & 7.1 & -18 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{T_{S1}} & -\frac{1}{T_{S1}} & 0\\ 0 & 0 & 0 & \frac{1}{T_{S2}} & 0 & -\frac{1}{T_{S2}}\end{pmatrix}$$

$$\boldsymbol{B} = 10^3\begin{pmatrix} 0 & 10^{-3} & 0\\ 0 & 0 & 10^{-3}\\ 8.1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \boldsymbol{B}_f = 10^3\begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 10^{-3}\\ 8.1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \boldsymbol{B_d} = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}, \quad \boldsymbol{C}_f = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$

and controlled by two decentralised PI-controllers.
After a blocking lower valve ($f : t > t_f u_L = 0$) at fault time t_f, the plant is statically I/O-reconfigurable according to Condition (2). The reconfiguration (3) yields

$$\boldsymbol{N} = \begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0.9167 & 1\end{pmatrix}.$$

The behaviour of the successfully reconfigured plant with fault f occurring at $t_f = 250$ s is shown in Figure 4. After the fault at $t_f = 250$ s and reconfiguration at $t = 260$ s, the control action

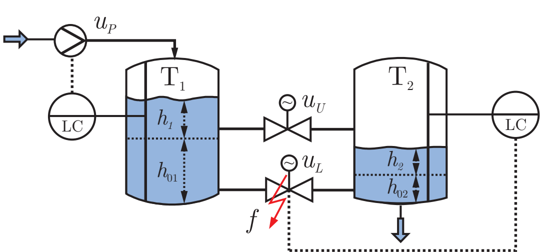

Figure 3: Laboratory two-tank system.

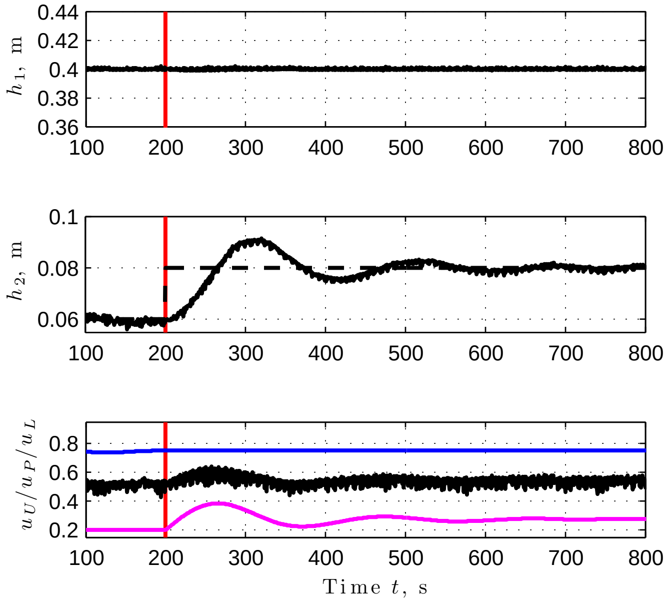

Figure 4: Reconfiguration after lower valve (u_L, blue) failure to upper valve (u_U, magenta).

is redirected from the lower to the upper valve. This action appears logical, but it is not found by the well-known pseudo-inverse method.
The approach was also successfully applied to a thermofluid process [6–8].

5 Generalised virtual actuator

In addition to its use as a stand-alone reconfiguration approach, the design for recovery of the I/O-behaviour can be used as part of a dynamic virtual actuator design approach. Its static block is designed to minimise the output correction of the virtual actuator for fault hiding. This approach was studied in [4].

References

[1] M. Blanke, M. Kinnaert, J. Lunze, and J. Staroswiecki. *Diagnosis and Fault-Tolerant Control*. Springer Verlag, Heidelberg, 2nd edition, 2006.

[2] J. Lunze and J. H. Richter. Reconfigurable fault-tolerant control: a tutorial introduction. *European J. Control*, 14(5):359–386, 2008.

[3] J. Lunze and J. H. Richter. Design approach to the generalised virtual actuator for control reconfiguration. *Automatisierungstechnik*, 54(7):353–361, July 2006.

[4] J. H. Richter, J. Lunze, and T. Schlage. Control reconfiguration after actuator failures by Markov parameter matching. *Int. J. Control*, 81(9):1382–98, September 2008.

[5] J. H. Richter and J. Lunze. Markov-parameter-based control reconfiguration by matching the I/O-behaviour of the plant. In *Proc. European Contr. Conf. (ECC) 2007*, pages 2942–49, Kos, Greece, July 2007. WeA11.3.

[6] J. H. Richter, T. Schlage, and J. Lunze. Rekonfiguration eines thermofluiden Prozesses mittels virtuellem Aktor. *Automatisierungstechnik*, 55(4):157–169, April 2007.

[7] J. H. Richter, T. Schlage, and J. Lunze. Control reconfiguration of a thermofluid process by means of a virtual actuator. *IET Control Theory Appl.*, 1(6):1606–20, November 2007.

[8] T. Schlage. Rekonfiguration einer Prozessregelung mittels virtuellem Aktor. Diplomarbeit, Lehrstuhl für Automatisierungstechnik und Prozessinformatik, Fakultät für Elektrotechnik und Informationstechnik, Ruhr-Universität Bochum, April 2006. ATP 0035.