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Event-based control of input-output linearizable systems

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1 Introduction

The main objective of event-based control is to reduce communication between sensors, controller and actuators. A data transfer is invoked only at time instants, when an internal trigger generates an event indicating that the control error has exceeded a tolerable threshold.

A novel structure of an event-based control loop shown in Fig. 1 has been introduced in [2]. Here the dashed arrow denotes a transmission of the current plant state $\boldsymbol{x}(t_k)$ from the event generator to the control input generator at the k-th event time t_k. The control input generator applies the received information to generate a control signal $\boldsymbol{u}(t)$, using a linear model of the plant. Having the same information, the event generator calculates the trajectory of the model state and compares it to the measured plant state $\boldsymbol{x}(t)$. A sufficiently large deviation between these values, caused by a disturbance $\boldsymbol{d}(t)$ affecting the plant, triggers an event. Disturbance rejection is the control aim, which makes feedback of information necessary [1].

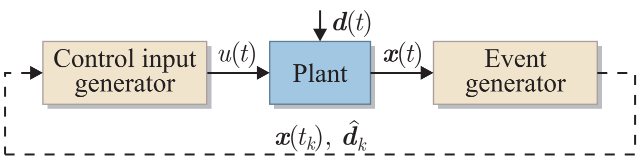

Figure 1: Event-based control loop

2 Project aims

The project aim is the extension of the above scheme to event-based control of nonlinear systems that are input-output linearizable. Following the idea presented in [2], the major design aim is to make the event-based control loop mimic a continuous state-feedback loop with prescribed accuracy.

The nonlinear plant that is investigated in this project is described by the affine state-space model

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_{\mathrm{x}}(\boldsymbol{x}(t)) + \boldsymbol{g}_{\mathrm{x}}(\boldsymbol{x}(t))u(t) + \boldsymbol{d}_{\mathrm{x}}(t) \tag{1}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_0$$

$$y(t) = h(\boldsymbol{x}(t)) \tag{2}$$

with single input and single output. The plant (1), (2) is assumed to be stable. With the output (2) the system is considered to have the relative degree $r = n$, thus, there exists a mapping $\boldsymbol{\phi} : \mathbb{R}^n \to \mathbb{R}^n$, $\boldsymbol{z}(t) = \boldsymbol{\phi}(\boldsymbol{x}(t))$ that transforms the system (1), (2) into normal form

$$\begin{aligned}\dot{\boldsymbol{z}}(t) &= \begin{pmatrix} z_2(t) \\ \vdots \\ z_n(t) \\ b(\boldsymbol{z}(t)) \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ a(\boldsymbol{z}(t)) \end{pmatrix} u(t) + \boldsymbol{d}(t) \\ &= \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}(\boldsymbol{z}(t))u(t) + \boldsymbol{d}(t) \qquad (3) \\ \boldsymbol{z}(0) &= \boldsymbol{z}_0 = \boldsymbol{\phi}(\boldsymbol{x}_0) \\ y(t) &= z_1(t)\end{aligned}$$

with the nonlinear functions $b(\boldsymbol{z}(t))$ and $a(\boldsymbol{z}(t))$ and $\boldsymbol{d}(t)$ being the transformed disturbance, [4].

In a further step the project extended the results to event-based control of nonlinear systems with internal dynamics for which $r < n$ holds, [5].

3 Nonlinear event-based control

A continuous state-feedback loop that serves as a *reference system* for the event-based control loop is obtained by applying to the plant (3) the state-feedback

$$u(t) = \frac{1}{a(\boldsymbol{z}(t))}\left(-b(\boldsymbol{z}(t)) - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{z}(t)\right), \tag{4}$$

which yields the linear system

$$\begin{aligned}\dot{\boldsymbol{z}}(t) &= \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & \dots & -k_n \end{pmatrix} \boldsymbol{z}(t) + \boldsymbol{d}(t) \\ &= \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{d}(t). \qquad (5)\end{aligned}$$

The static state-feedback gain $\boldsymbol{k}^{\mathrm{T}}$ is designed such that the continuous control loop (5) is stable and has a satisfactory disturbance rejection behavior.

The *control input generator* uses a copy of the model (5) with model state $\boldsymbol{z}_{\mathrm{s}}$

$$\dot{\boldsymbol{z}}_{\mathrm{s}}(t) = \boldsymbol{A}\boldsymbol{z}_{\mathrm{s}}(t) + \hat{\boldsymbol{d}}_k, \quad \boldsymbol{z}_{\mathrm{s}}(t_k^+) = \boldsymbol{z}(t_k) \tag{6}$$

for the time interval $t \in [t_k, t_k + 1)$, where $\hat{\boldsymbol{d}}_k$ is an estimation of the disturbance $\boldsymbol{d}(t)$, in order to generate a control input $u(t)$ according to Eqn. (4)

$$u(t) = \frac{1}{a(\boldsymbol{z}_{\mathrm{s}}(t))}\left(-b(\boldsymbol{z}_{\mathrm{s}}(t)) - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{z}_{\mathrm{s}}(t)\right). \tag{7}$$

The model (6) is reinitialized with the current plant state $\boldsymbol{z}(t_k)$ at the time t_k^+ that denotes the time instant right after the event time t_k.

The plant (3) together with the control input (7) yields the model

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{e}_1 \mu(\boldsymbol{z}(t), \boldsymbol{z}_{\mathrm{s}}(t)) + \boldsymbol{d}(t), \quad \boldsymbol{z}(0) = \boldsymbol{z}_0$$

with the n-dimensional vector $\boldsymbol{e}_1 = \begin{pmatrix} 0 & \dots & 0 & 1 \end{pmatrix}^{\mathrm{T}}$ and the nonlinear term

$$\mu(\boldsymbol{z}, \boldsymbol{z}_{\mathrm{s}}) = \left(b(\boldsymbol{z}) - \frac{a(\boldsymbol{z})}{a(\boldsymbol{z}_{\mathrm{s}})} b(\boldsymbol{z}_{\mathrm{s}}) \right) + \boldsymbol{k}^{\mathrm{T}} \left(\boldsymbol{z} - \frac{a(\boldsymbol{z})}{a(\boldsymbol{z}_{\mathrm{s}})} \boldsymbol{z}_{\mathrm{s}} \right), \tag{8}$$

which is zero when the plant state $\boldsymbol{z}(t)$ and the model state $\boldsymbol{z}_{\mathrm{s}}(t)$ coincide, but is not equal to zero if the states $\boldsymbol{z}(t)$ and $\boldsymbol{z}_{\mathrm{s}}(t)$ diverge. The *event generator* limits the growth of the term (8) by initiating a communication to the control input generator each time when

$$|\mu(\boldsymbol{z}(t_{k+1}), \boldsymbol{z}_{\mathrm{s}}(t_{k+1}))| = \bar{e} \tag{9}$$

is satisfied, with $\bar{e} \in \mathbb{R}_+$ as the event generation threshold. At time t_{k+1}^+ the model of the control input generator is reset with the state $\boldsymbol{z}(t_{k+1})$ and, hence,

$$\left|\mu(\boldsymbol{z}(t_{k+1}^+), \boldsymbol{z}_{\mathrm{s}}(t_{k+1}^+))\right| = 0$$

holds.

The *disturbance estimator* is implemented in the event generator and uses the model

$$\dot{\boldsymbol{z}}_{\mathrm{e}}(t) = \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}(\boldsymbol{z}(t)) u(t), \quad \boldsymbol{z}_{\mathrm{e}}(t_k) = \boldsymbol{z}(t_k)$$

to determine the estimation $\hat{\boldsymbol{d}}_{k+1}$ by

$$\hat{\boldsymbol{d}}_{k+1} = \frac{1}{t_{k+1} - t_k} \left(\boldsymbol{z}(t_{k+1}) - \boldsymbol{z}_{\mathrm{e}}(t_{k+1}) \right)$$

that is used for $t \geq t_{k+1}$.

4 Analysis results

As the main analysis result, the difference between the plant state $\boldsymbol{z}(t)$ of the event-based control loop and the state $\boldsymbol{z}_{\mathrm{SF}}(t)$ of the reference system

$$\dot{\boldsymbol{z}}_{\mathrm{SF}}(t) = \boldsymbol{A}\boldsymbol{z}_{\mathrm{SF}}(t) + \boldsymbol{d}(t), \quad \boldsymbol{z}_{\mathrm{SF}}(0) = \boldsymbol{z}_0$$

is proved to be bounded according to

$$||\boldsymbol{\delta}(t)|| = ||\boldsymbol{z}(t) - \boldsymbol{z}_{\mathrm{SF}}(t)|| \leq \delta_{\max}$$

with

$$\delta_{\max} = \bar{e} \cdot \int_0^\infty \left|\left| \mathrm{e}^{\boldsymbol{A}\tau} \boldsymbol{e}_1 \right|\right| \mathrm{d}\tau.$$

The second investigation concerns the communication frequency of the event-based control scheme and uses the value ζ as the minimal deviation between the plant state $\boldsymbol{z}$ and the model state $\boldsymbol{z}_{\mathrm{s}}$ for which the condition (9) is satisfied:

$$\zeta := \min_{\boldsymbol{z}, \boldsymbol{z}_{\mathrm{s}}} ||\boldsymbol{z} - \boldsymbol{z}_{\mathrm{s}}||, \quad \text{s.t.} \ |\mu(\boldsymbol{z}, \boldsymbol{z}_{\mathrm{s}})| = \bar{e}.$$

For a bounded disturbance

$$||\boldsymbol{d}_\Delta(t)|| = \left|\left| \boldsymbol{d}(t) - \hat{\boldsymbol{d}}_k \right|\right| \leq d_{\Delta\max}$$

the event-based control loop is shown to have a minimal inter-event time $T_{\min}$ that is bounded from below by

$$T_{\min} \geq \overline{T}$$

with $\overline{T}$ satisfying

$$\int_0^{\overline{T}} \left|\left| \mathrm{e}^{\boldsymbol{A}\tau} \right|\right| \mathrm{d}\tau = \frac{\zeta}{\bar{e} + d_{\Delta\max}}.$$

5 Experimental example

The event-based control approach shall be applied to control a chemical reaction in a continuous stirred tank reactor (Fig. 2). The tank is fed by a reactant A with temperature ϑ_{in} and concentration c_{in}. The temperature ϑ_{c} of the cooling jacket is affected by the cooling power $\dot{Q}$. The liquid in the tank is supposed to be at a constant level. The reactions inside the liquid are described by the "van de Vusse" reaction scheme

$$A \rightarrow B \rightarrow C, \quad 2A \rightarrow D,$$

comprising the reaction of educt A to the desired product B and the parallel reactions to the byproducts C and D.

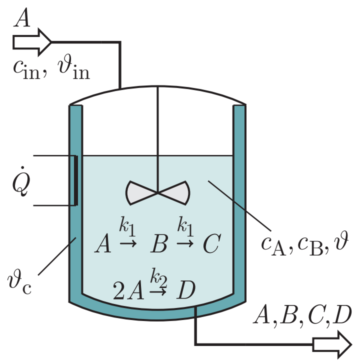

Figure 2: Scheme of the continuous stirred tank reactor

The control aim is to keep the concentrations c_{A} and c_{B} of the reactant A and the product B constant. A disturbance of the process is realized by a variation of the cooling power $\dot{Q}$ that has an impact on the chemical reaction via the temperature ϑ.

References

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