

## Controller Synthesis for a Class of Hybrid Systems

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### 1 Introduction

Hybrid systems are dynamical systems which are characterized by the interaction of continuous-variable and discrete-variable dynamics (Figure 1). These systems typically contain signals which take values from a continuous set (typically the set of real numbers), and also signals which have discrete values (e.g. the set of symbols  $\{a, b, c\}$ ). Examples of such systems can be found in automotive control, robotics, interconnected power systems, automated manufacturing and process control systems, where continuous systems are controlled by discrete controllers.

The last decade has seen a dramatic increase in research on hybrid systems, but a lot still remains to be done in the areas of model-building, analysis and controller design. The main difficulty in the research on hybrid systems arises from the fact that the theories of continuous dynamics and discrete dynamics have been developed separately, with control engineers on one hand developing theories for continuous systems based on differential equations, and computer scientists on the other hand developing theories for discrete-event systems using automata. It is well known that one cannot deduce the properties of a hybrid system by considering the discrete subsystem and the continuous subsystem separately, and an efficacious framework which considers both types of dynamics simultaneously is yet to be developed. As a result, different models have been proposed for describing hybrid systems, [1].

### 2 Project Aim

This project deals with the controller design for hybrid systems. The basic problem is stated as follows:

**Given:** A hybrid system with

- discrete dynamics described using deterministic finite automata
- linear continuous dynamics where some states may not be controllable,

**Design:** A hybrid controller that automatically drives the hybrid system to different discrete states so as to move the continuous state from a given initial state set to a target state set while meeting some performance criteria.

To systematically solve this problem, it is broken down into the subtasks listed in the following sections.

### 3 Hybrid model selection

Although general models of hybrid systems can represent many of the characteristics of the hybrid systems and can model a large number of systems, they are usually too complicated for use in analysis and controller design. For this work, the simplified model structure shown in Figure 1 has been formulated, where  $x$  is the continuous state vector and  $z$  is the discrete state vector.

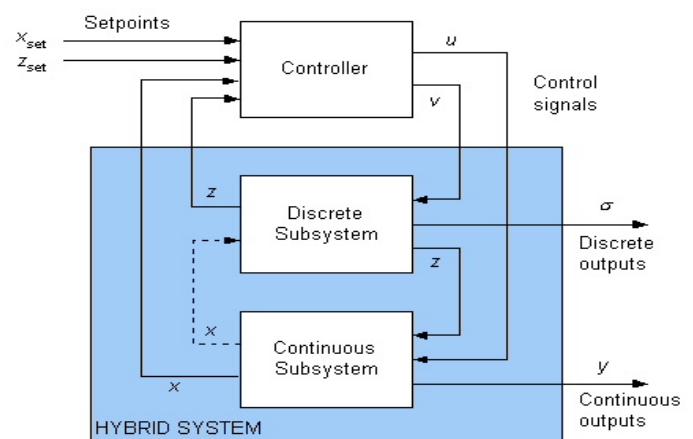


Figure 1: Structure of hybrid control system

This simplified model can represent a wide variety of systems, in particular hybrid systems characterized by hybrid phenomena of *controlled switching*. Examples of such systems include chemical plants and gearbox systems. For these systems, the discrete state can be directly prescribed by the controller.

## 4 Analysis

### 4.1 Controllability Analysis

The following questions need to be answered in the context of the given model:

- How does one define and test for controllability in hybrid systems?
- Given a hybrid system where the continuous subsystems are uncontrollable, can one systematically derive a way of switching between the different continuous subsystems so that the hybrid system is controllable?

### 4.2 Reachability Analysis

The issues considered in the reachability analysis of hybrid systems are *computability* (representation of sets, computation of successor (or predecessor) states, union and/or intersection of sets and emptiness checking), and *finiteness* (whether the algorithm is guaranteed to terminate in a finite number of steps, also known as decidability). For hybrid systems, the reachable sets for both the discrete and the continuous dynamics have to be computed, and the difficulty here arises in the computation of reach sets for non-trivial continuous dynamics. There are many approaches for computing the reachable sets for continuous-time systems, mainly drawn from optimal control, game theory and computational geometry. These approaches consider the reachable sets in the full state dimension, leading to complicated data structures for representing the reach sets, which makes these methods difficult to apply. An alternative way of reachability analysis has been developed in [3] where the whole hybrid system is considered as a purely discrete-event system and the discrete reachability has been analyzed. This approach reduces the complexity of the analysis considerably.

To simplify the reachability analysis in this work, the initial and final continuous states are specified as thresholds in the state-space, not as regions, and this simplifies the analysis because it is easier to check if a threshold has been crossed than to check if the reachable region includes a certain region. Furthermore, this approach does not require the symbolic representation of the reach set.

### 4.3 Trajectory Determination

Using the results from reachability analysis above, a suitable trajectory from the initial state to the final state can be found. To carry out the control task, the system state will have to be guided along this trajectory. The trajectory is to be chosen in such a way that the system gets to the target state as fast as possible, avoids ‘forbidden’ states, excludes the occurrence of deadlocks and Zeno behaviour, and meets some robustness criteria.

## 5 Controller Design

A controller has to guide the system through the trajectory determined above. The controller gives both continuous and discrete control signals, hence, it is a hybrid controller. To design the controller, the control task will be decomposed into continuous and discrete subtasks. Suitable control structures for the continuous and discrete parts of the controller (and any necessary interfaces between them) are then to be determined.

## 6 Experimental work

The applicability of the theoretical results obtained in the research work shall be demonstrated on a Manufacturing Cell with hybrid characteristics that is located in the Institute of Automation and Computer Control at the Ruhr University Bochum (see Figure 2).

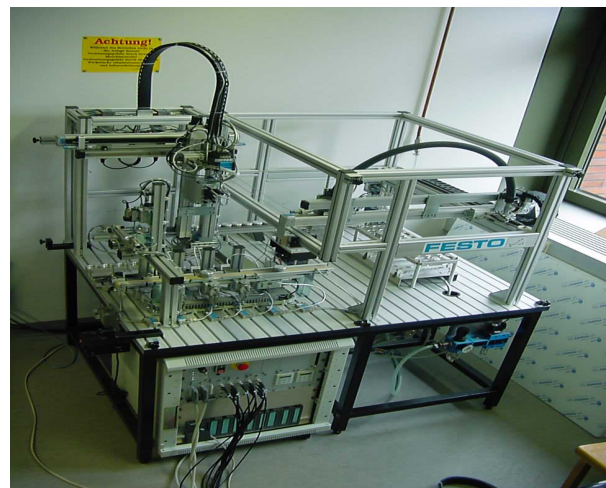


Figure 2: The Manufacturing Cell

The Manufacturing Cell is an electro-pneumatically-driven experimental model of a manufacturing process. It is controlled using a Programmable Logic Controller (PLC) and a Digital Signal Processor (DSP). For this particular Manufacturing Cell, workpieces in the form of cylindrical aluminium blocks can be heated and placed in different positions within the plant. The state of a workpiece is defined using a continuous-valued temperature and a discrete-valued position. The continuous-valued states depend on the discrete-valued states, hence the dynamics of the Manufacturing Cell are hybrid in nature.

## References

- [1] S. KAMAU, *Different Approaches to Modelling of Hybrid Systems*, tech. rep., Lehrstuhl für Automatisierungstechnik und Prozessinformatik, Ruhr-Universität Bochum, 2001.
- [2] S. KAMAU AND G. LICHTENBERG, *A Classification of the Input-Output Behaviour of Hybrid Systems*, in Proceedings of the European Control Conference, 2001, pp. 3143–3148.
- [3] J. LUNZE AND B. NIXDORF, *Discrete Reachability of Hybrid Systems*, tech. rep., Lehrstuhl für Automatisierungstechnik und Prozessinformatik, Ruhr-Universität Bochum, 2001.