Distributed control of interconnected systems with event-based information requests ⋆

Christian Stöcker * **and Jan Lunze** *

* *Ruhr-Universität Bochum, 44780 Bochum, Germany,*
(e-mail: {stoecker, lunze}@atp.rub.de)

Abstract: This paper proposes a new approach to distributed control of physically interconnected subsystems which combines continuous and event-based state feedback. The main aim of the controller is to suppress the propagation of a disturbance within an interconnection of subsystems. The novelty of this approach is that the controllers request current state information from the neighboring systems at the event times. Events are generated based on a condition for which only local information are applied. The disturbance rejection behavior of the control approach with event-based information requests is demonstrated in an illustrative example which shows that the disturbance propagation is considerably reduced compared to a continuous decentralized state-feedback controller.

Keywords: Distributed control, Event-based control, Networked control system, Information requests

1. INTRODUCTION

1.1 Control with event-based requests: Basic idea

The aim of event-based control is to restrict the communication among components of a control system to time instants at which the exchange of current information is necessary to ensure a desired behavior of the closed-loop system. This paper studies the event-based disturbance rejection of N physically interconnected linear subsystems (Fig. 1) and it introduces a new kind of distributed control which combines continuous and event-based state feedback. The novelty of this approach is that the controllers trigger events if they need information from the neighboring systems. Therefore, at the event times the controllers send a request to the neighboring systems to transmit their current state.

The basic idea of the proposed control method is as follows: The controller $\mathcal{C}_i$ of subsystem Σ_i generates the control input $\boldsymbol{u}_i(t)$ according to a distributed control law using the local state $\boldsymbol{x}_i(t)$ as well as estimates $\tilde{\boldsymbol{x}}_j(t)$ of the states $\boldsymbol{x}_j(t)$ of the neighboring subsystems Σ_j of subsystem Σ_i. A deviation between the states $\boldsymbol{x}_j(t)$ and their estimates $\tilde{\boldsymbol{x}}_j(t)$ occurs due to disturbances that affect the overall control system. This influence of the disturbances is monitored in $\mathcal{C}_i$ by comparing the measured coupling input $\boldsymbol{s}_i(t)$ with its estimate $\tilde{\boldsymbol{s}}_i(t)$. If at some time t_{k_i} the deviation between $\boldsymbol{s}_i(t_{k_i})$ and $\tilde{\boldsymbol{s}}_i(t_{k_i})$ exceeds a tolerable bound, $\mathcal{C}_i$ requests the current states $\boldsymbol{x}_j(t_{k_i})$ from the neighboring subsystems Σ_j which are used in Σ_i to update the estimation.

⋆ This work was supported by the German Research Foundation (DFG) within the Priority Program 1305 "Control Theory of Digitally Networked Dynamical Systems".

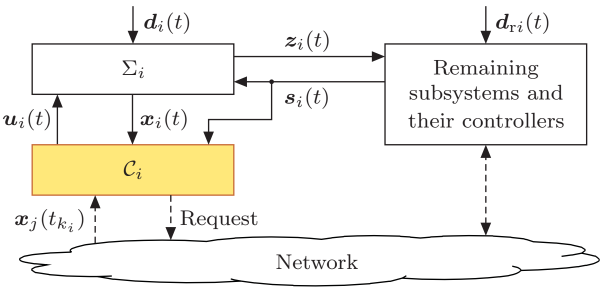

Fig. 1. Structure of the control system from the viewpoint of subsystem Σ_i

1.2 Literature review

Decentralized or distributed event-based control has been investigated in several publications, e. g. by Dimarogonas et al. (2012); Seyboth et al. (2013) regarding event-based multi-agent control or by Mazo Jr. and Tabuada (2011); De Persis et al. (2013); Stöcker et al. (2012); Wang and Lemmon (2011); Yook et al. (2002) regarding the event-based stabilization of interconnected subsystems. Li and Lemmon (2011); Donkers and Heemels (2012) studied control of systems that have separate links between the sensors and the controller and between the controller and the actuators and which are used asynchronously in an event-based fashion for the stabilization of the system. In the existing literature that deals with distributed event-based control the control input is kept constant in between consecutive events. This differs from this paper where a model-based approach to event-based control is proposed following the idea of Lunze and Lehmann (2010).

All the above cited references have in common that the triggering of an event causes the transmission of current information from the component which has triggered the event to some other component. In contrast to this, the present work proposes a method where the controllers perceive that current state information is required to ensure a desired disturbance rejection behavior. Therefore, the alerting controller requests at the event times state information from the neighboring subsystems. This is a new type of event-based control which has not been published before.

1.3 Outline

Section 2 introduces a decomposition of the overall control system into different models which is used for the design method of the distributed controller with event-based information requests presented in Sec. 3. The stability of the overall control system is investigated in Sec. 4 where the main analysis result is presented in Theorem 8. Section 5 gives an illustrative example which highlights the fact that the novel control approach with event-based information requests considerably reduced the disturbance propagation across the interconnected subsystems compared to a continuous decentralized state-feedback control.

1.4 Preliminaries

The following notation will be used. $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of real numbers and positive real numbers, respectively. $\mathbb{N}_0$ refers to the set of natural numbers including 0. For a matrix $\boldsymbol{M}$ and a vector $\boldsymbol{v}$, $\|\boldsymbol{M}\|$, $\|\boldsymbol{v}\|$ denote an arbitrary matrix norm and its induced vector norm, respectively. The asterisk $*$ represents the convolution-operator, e. g.

$$\boldsymbol{G} * \boldsymbol{u} = \int_0^t \boldsymbol{G}(t-\tau)\boldsymbol{u}(\tau)\mathrm{d}\tau.$$

$\boldsymbol{A} = \mathrm{diag}\,(\boldsymbol{A}_1, ..., \boldsymbol{A}_N)$ is a block diagonal matrix with the matrices $\boldsymbol{A}_i$ $(i = 1, \ldots, N)$ on the main diagonal.

Consider a system the behavior of which from input $\boldsymbol{u}(t)$ to output $\boldsymbol{y}(t)$ is described by the relation

$$\boldsymbol{y}(t) = \boldsymbol{G} * \boldsymbol{u} \tag{1}$$

with the impulse response matrix $\boldsymbol{G}(t)$.

Definition 1. (Sontag (1998)). The system (1) is said to be *uniformly bounded-input bounded-output stable* (UBIBO-stable) if its impulse response $\boldsymbol{G}(t)$ is integrable.

In this paper event-based control systems are investigated which can be modeled as an impulsive system

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{E}\boldsymbol{d}(t), \quad \text{if } \boldsymbol{x}(t) \in \mathcal{F} \tag{2a}$$

$$\boldsymbol{x}(t^+) = \boldsymbol{G}\boldsymbol{x}(t), \quad \text{if } \boldsymbol{x}(t) \in \mathcal{R} \tag{2b}$$

where $\mathcal{F}$ and $\mathcal{R}$ are referred to as the flow set and reset set, respectively (cf. Donkers and Heemels (2012); Stöcker and Lunze (2013)). Hence, the systems that are investigated here belong to a special class of hybrid dynamical systems and, therefore, require a different notion of stability. In the following the concept of *ultimate boundedness* is used.

Definition 2. (Khalil (2002)). The solution $\boldsymbol{x}(t)$ of the system (2) is globally uniformly ultimately bounded (GUUB) if for every $\boldsymbol{x}(0) \in \mathbb{R}^n$ there exists a constant $p \in \mathbb{R}_+$ and a time $\bar{t}$ such that

$$\boldsymbol{x}(t) \in \Omega := \{\boldsymbol{x} \mid \|\boldsymbol{x}\| \le p\}, \quad \forall\, t \ge \bar{t}$$

holds. The system (2) is said to be ultimately bounded if its state $\boldsymbol{x}(t)$ is GUUB.

2. MODELS

This section introduces the notion of the *extended subsystem model* $\Sigma_{\mathrm{e}i}$, which consists of subsystem Σ_i augmented with model information of some other subsystems Σ_j. This model representation is the basis for the new approach to state-feedback control with event-based information requests introduced in Sec. 3.

2.1 Subsystem model

The i-th subsystem is represented by the state-space model

$$\Sigma_i : \begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}_i\boldsymbol{x}_i(t) + \boldsymbol{B}_i\boldsymbol{u}_i(t) + \boldsymbol{E}_i\boldsymbol{d}_i(t) + \boldsymbol{E}_{\mathrm{s}i}\boldsymbol{s}_i(t) \\ \boldsymbol{x}_i(0) = \boldsymbol{x}_{0i} \\ \boldsymbol{z}_i(t) = \boldsymbol{C}_{\mathrm{z}i}\boldsymbol{x}_i(t) \end{cases} \tag{3}$$

where $\boldsymbol{x}_i \in \mathbb{R}^{n_i}$, $\boldsymbol{u}_i \in \mathbb{R}^{m_i}$, $\boldsymbol{s}_i \in \mathbb{R}^{p_i}$, $\boldsymbol{z}_i \in \mathbb{R}^{q_i}$ denote the state, control input, coupling input and coupling output of Σ_i, respectively. The disturbance $\boldsymbol{d}_i \in \mathbb{R}^{w_i}$ is assumed to be bounded by some bound $\hat{d}_i \in \mathbb{R}_+$:

$$\|\boldsymbol{d}_i(t)\| \le \hat{d}_i, \quad \forall\, t \ge 0. \tag{4}$$

Concerning the subsystem Σ_i the following assumption is made:

Assumption 3. For each $i \in \mathcal{N} := \{1, \ldots, N\}$ the pair $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ is controllable and both the state $\boldsymbol{x}_i(t)$ and the coupling input $\boldsymbol{s}_i(t)$ are measurable.

The subsystems (3) are interconnected according to the relation

$$\boldsymbol{s}_i(t) = \sum_{j=1}^{N} \boldsymbol{L}_{ij}\boldsymbol{z}_j(t), \quad \forall\, i \in \mathcal{N}. \tag{5}$$

Subsystems which are directly interconnected are called *neighbors*:

Definition 4. Σ_j is called neighbor of Σ_i, if $\|\boldsymbol{L}_{ij}\| > 0$ holds. In the following,

$$\mathcal{N}_i := \{j \mid \|\boldsymbol{L}_{ij}\| > 0\} \subseteq \mathcal{N} \setminus \{i\}$$

is referred to as the set of neighbors of Σ_i which contains the numbers of those subsystems, which are directly interconnected with Σ_i.

2.2 Approximate model and residual model

Consider subsystem Σ_i defined in (3). According to (5) the coupling input $\boldsymbol{s}_i(t)$ aggregates the influence of the remaining subsystems together with their controllers on Σ_i (Fig. 2(a)). Assume that, from the viewpoint of Σ_i, the relation between the coupling output $\boldsymbol{z}_i(t)$ and the coupling input $\boldsymbol{s}_i(t)$ is approximately described by some *approximate model*

$$\Sigma_{\mathrm{a}i} : \begin{cases} \dot{\boldsymbol{x}}_{\mathrm{a}i}(t) = \boldsymbol{A}_{\mathrm{a}i}\boldsymbol{x}_{\mathrm{a}i}(t) + \boldsymbol{B}_{\mathrm{a}i}\boldsymbol{z}_i(t) + \boldsymbol{E}_{\mathrm{a}i}\boldsymbol{d}_{\mathrm{a}i}(t) \\ \qquad\qquad + \boldsymbol{F}_{\mathrm{a}i}\boldsymbol{f}_i(t) \\ \boldsymbol{x}_{\mathrm{a}i}(0) = \boldsymbol{x}_{\mathrm{a}0i} \\ \boldsymbol{s}_i(t) = \boldsymbol{C}_{\mathrm{a}i}\boldsymbol{x}_{\mathrm{a}i}(t) \\ \boldsymbol{v}_i(t) = \boldsymbol{H}_{\mathrm{a}i}\boldsymbol{x}_{\mathrm{a}i}(t), \end{cases} \tag{6}$$

where $\boldsymbol{x}_{\mathrm{a}i} \in \mathbb{R}^{n_{\mathrm{a}i}}, \boldsymbol{d}_{\mathrm{a}i} \in \mathbb{R}^{w_{\mathrm{a}i}}, \boldsymbol{f}_i \in \mathbb{R}^{r_i}$ and $\boldsymbol{v}_i \in \mathbb{R}^{s_i}$ denote the state, the disturbance, the residual output and residual input, respectively. The disturbance $\boldsymbol{d}_{\mathrm{a}i}(t)$ is considered to satisfy the relation

$$\|\boldsymbol{d}_{\mathrm{a}i}(t)\| \le \hat{d}_{\mathrm{a}i}, \quad \forall\, t \ge 0 \tag{7}$$

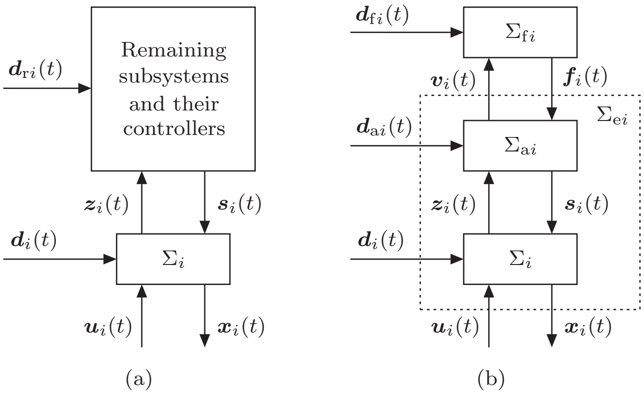

Fig. 2. Interconnection of subsystem Σ_i and the remaining control loops: (a) general structure; (b) decomposition of the remaining controlled subsystems into approximate model $\Sigma_{\mathrm{a}i}$ and residual model $\Sigma_{\mathrm{f}i}$.

for some finite $\hat{d}_{\mathrm{a}i} \in \mathbb{R}_+$. The approximate model $\Sigma_{\mathrm{a}i}$ is assumed to have the following properties.

Assumption 5. The matrix $\boldsymbol{A}_{\mathrm{a}i}$ is Hurwitz and the pair $(\boldsymbol{A}_{\mathrm{a}i}, \boldsymbol{C}_{\mathrm{a}i})$ is observable.

The mismatch between the behavior of the remaining controlled subsystems and the approximate model (6) is expressed by the *residual model*

$$\Sigma_{\mathrm{f}i} : \boldsymbol{f}_i(t) = \boldsymbol{G}_{\mathrm{fd}i} * \boldsymbol{d}_{\mathrm{f}i} + \boldsymbol{G}_{\mathrm{fv}i} * \boldsymbol{v}_i \tag{8}$$

where $\boldsymbol{d}_{\mathrm{f}i} \in \mathbb{R}^{w_{\mathrm{f}i}}$ denotes the disturbance on $\Sigma_{\mathrm{f}i}$ that is considered to be bounded:

$$\|\boldsymbol{d}_{\mathrm{f}i}(t)\| \le \hat{d}_{\mathrm{f}i}, \quad \forall\, t \ge 0. \tag{9}$$

Hence, the approximate model $\Sigma_{\mathrm{a}i}$ together with the residual model $\Sigma_{\mathrm{f}i}$ represents the behavior of the remaining subsystems and their controllers (Fig. 2(b)). Hereafter the residual model $\Sigma_{\mathrm{f}i}$ is not assumed to be known exactly but described by some upper bounds $g_{\mathrm{fd}i}(t)$ and $g_{\mathrm{fv}i}(t)$:

$$g_{\mathrm{fd}i}(t) \ge \|\boldsymbol{G}_{\mathrm{fd}i}(t)\|, \quad g_{\mathrm{fv}i}(t) \ge \|\boldsymbol{G}_{\mathrm{fv}i}(t)\|, \quad \forall\, t \ge 0. \tag{10}$$

In the following, it is assumed that the state $\boldsymbol{x}_{\mathrm{a}i}(t)$ of the approximate model $\Sigma_{\mathrm{a}i}$ is related with the states $\boldsymbol{x}_j(t)$ ($j \in \mathcal{N}_i$) of the neighboring subsystems of Σ_i. Hence, the approximate model $\Sigma_{\mathrm{a}i}$ can be obtained by the linear transformation

$$\boldsymbol{x}_{\mathrm{a}i}(t) = \sum_{j \in \mathcal{N}_i} \boldsymbol{T}_{ij} \boldsymbol{x}_j(t), \quad \forall\, t \ge 0 \tag{11}$$

with $\boldsymbol{T}_{ij} \in \mathbb{R}^{n_{\mathrm{a}i} \times n_j}$. Appendix A presents an example that shows how the approximate model can be determined using the transformation (11) for a system that is composed of serially interconnected subsystems.

2.3 Extended subsystem model

Consider the subsystem Σ_i augmented with the approximate model $\Sigma_{\mathrm{a}i}$ which yields *extended subsystem*

$$\Sigma_{\mathrm{e}i} : \begin{cases} \dot{\boldsymbol{x}}_{\mathrm{e}i}(t) = \boldsymbol{A}_{\mathrm{e}i}\boldsymbol{x}_{\mathrm{e}i}(t) + \boldsymbol{B}_{\mathrm{e}i}\boldsymbol{u}_i(t) + \boldsymbol{E}_{\mathrm{e}i}\boldsymbol{d}_{\mathrm{e}i}(t) \\ \qquad\qquad + \boldsymbol{F}_{\mathrm{e}i}\boldsymbol{f}_i(t) \\ \boldsymbol{x}_{\mathrm{e}i}(0) = \left(\boldsymbol{x}_{0i}^\top \ \boldsymbol{x}_{\mathrm{a}0i}^\top\right)^\top \\ \boldsymbol{v}_i(t) = \boldsymbol{H}_{\mathrm{e}i}\boldsymbol{x}_{\mathrm{e}i}(t) \end{cases} \tag{12}$$

with the state $\boldsymbol{x}_{\mathrm{e}i} = (\boldsymbol{x}_i^\top \ \boldsymbol{x}_{\mathrm{a}i}^\top)^\top$, the composite disturbance vector $\boldsymbol{d}_{\mathrm{e}i} = (\boldsymbol{d}_i^\top \ \boldsymbol{d}_{\mathrm{a}i}^\top)^\top$ and the matrices

$$\boldsymbol{A}_{\mathrm{e}i} = \begin{pmatrix} \boldsymbol{A}_i & \boldsymbol{E}_{\mathrm{s}i}\boldsymbol{C}_{\mathrm{a}i} \\ \boldsymbol{B}_{\mathrm{a}i}\boldsymbol{C}_{\mathrm{z}i} & \boldsymbol{A}_{\mathrm{a}i} \end{pmatrix}, \quad \boldsymbol{B}_{\mathrm{e}i} = \begin{pmatrix} \boldsymbol{B}_i \\ \boldsymbol{O} \end{pmatrix},$$

$$\boldsymbol{E}_{\mathrm{e}i} = \begin{pmatrix} \boldsymbol{E}_i & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{E}_{\mathrm{a}i} \end{pmatrix}, \quad \boldsymbol{F}_{\mathrm{e}i} = \begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{F}_{\mathrm{a}i} \end{pmatrix}, \quad \boldsymbol{H}_{\mathrm{e}i} = (\boldsymbol{O} \ \boldsymbol{H}_{\mathrm{a}i}).$$

Given Eqs. (4), (7), the augmented disturbance $\boldsymbol{d}_{\mathrm{e}i}(t)$ is bounded from above by

$$\|\boldsymbol{d}_{\mathrm{e}i}(t)\| \le \hat{d}_i + \hat{d}_{\mathrm{a}i} =: \hat{d}_{\mathrm{e}i}, \quad \forall\, t \ge 0. \tag{13}$$

The next section presents an event-based controller which is designed on the basis of the extended subsystem model $\Sigma_{\mathrm{e}i}$.

3. DISTRIBUTED CONTROL WITH EVENT-BASED INFORMATION REQUESTS

This section introduces a new event-based control method for the disturbance rejection for physically interconnected subsystems. The presented event-based controller of subsystem Σ_i requests at the event times t_{k_i} information from the neighbor subsystems Σ_j ($j \in \mathcal{N}_i$). The controller works in a distributed manner and it uses both local state information $\boldsymbol{x}_i(t)$ and information from interconnected subsystems Σ_j. The proposed control strategy using *information requests* leads to a new kind of event-based control which contrasts with almost all event-based control approaches in the existing literature, where events trigger the sending of information.

3.1 Control aim

The aim of the distributed controller of Σ_i is

(1) to attenuate the disturbance $\boldsymbol{d}_i(t)$ on Σ_i and
(2) to reduce influence of the interconnected subsystems on subsystem Σ_i compared to a decentralized continuous state-feedback control.

Assume that this aim is accomplished by means of the control law

$$\boldsymbol{u}_i(t) = \underbrace{-\boldsymbol{K}_i\boldsymbol{x}_i(t)}_{:= \boldsymbol{u}_{\mathrm{d}i}(t)} \ \underbrace{-\boldsymbol{K}_{\mathrm{a}i}\boldsymbol{x}_{\mathrm{a}i}(t)}_{:= \boldsymbol{u}_{\mathrm{a}i}(t)} \tag{14}$$

where $\boldsymbol{x}_{\mathrm{a}i}(t)$ is the state of the approximate model $\Sigma_{\mathrm{a}i}$ and the gains $\boldsymbol{K}_i$ and $\boldsymbol{K}_{\mathrm{a}i}$ are appropriately chosen in a sense that is specified later. The controller of Σ_i has no permanent access to the approximate model state $\boldsymbol{x}_{\mathrm{a}i}(t)$ and, hence, Eq. (14) is not applicable.

The basic idea how the control aim can be achieved nonetheless is to imitate the continuous state-feedback (14) by an event-based controller that is introduced in the next section.

3.2 Event-based controller

The event-based controller $\mathcal{C}_i$ that is presented in this section is assumed to have continuous access to the subsystem state $\boldsymbol{x}_i(t)$ and the coupling input signal $\boldsymbol{s}_i(t)$. Thus, $\mathcal{C}_i$ can determine the first part of the control law (14) denoted by $\boldsymbol{u}_{\mathrm{d}i}(t)$ in a continuous manner, whereas the part $\boldsymbol{u}_{\mathrm{a}i}(t)$ cannot be implemented as stated in (14). The following explains how the controller $\mathcal{C}_i$ approximates the signal $\boldsymbol{u}_{\mathrm{a}i}(t)$ and requests current state information from the neighboring subsystems if needed.

The controller $\mathcal{C}_i$ determines an approximation $\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \in \mathbb{R}^{n_{\mathrm{a}i}}$ of the current state $\boldsymbol{x}_{\mathrm{a}i}(t)$ using the model

$$\tilde{\Sigma}_{\mathrm{a}i}: \begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) = \boldsymbol{A}_{\mathrm{a}i}\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) + \boldsymbol{B}_{\mathrm{a}i}\boldsymbol{C}_{\mathrm{z}i}\boldsymbol{x}_i(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t_{k_i}^+) = \sum_{j \in \mathcal{N}_i} \boldsymbol{T}_{ij}\boldsymbol{x}_j(t_{k_i}) \\ \tilde{\boldsymbol{s}}_i(t) = \boldsymbol{C}_{\mathrm{a}i}\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{cases} \tag{15}$$

where the relation $\boldsymbol{z}_i(t) = \boldsymbol{C}_{\mathrm{z}i}\boldsymbol{x}_i(t)$ is applied. Having the approximation $\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t)$, the controller generates the control input $\boldsymbol{u}_i(t)$ according to the control law

$$\boldsymbol{u}_i(t) = -\boldsymbol{K}_i\boldsymbol{x}_i(t) - \boldsymbol{K}_{\mathrm{a}i}\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t). \tag{16}$$

The feedback-gains $\boldsymbol{K}_i$ and $\boldsymbol{K}_{\mathrm{a}i}$ are assumed to be designed such that the matrix

$$\bar{\boldsymbol{A}}_{\mathrm{e}i} := \begin{pmatrix} \boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_i & \boldsymbol{E}_{\mathrm{s}i}\boldsymbol{C}_{\mathrm{a}i} - \boldsymbol{B}_i\boldsymbol{K}_{\mathrm{a}i} \\ \boldsymbol{B}_{\mathrm{a}i}\boldsymbol{C}_{\mathrm{z}i} & \boldsymbol{A}_{\mathrm{a}i} \end{pmatrix} \tag{17}$$

is Hurwitz.

In general, the approximate state $\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t)$ deviates from the current state $\boldsymbol{x}_{\mathrm{a}i}(t)$, since in the model (15) the influences of the disturbance $\boldsymbol{d}_{\mathrm{a}i}(t)$ and of the residual model via the signal $\boldsymbol{f}_i(t)$ are omitted. In order to bound the deviation between the state $\boldsymbol{x}_{\mathrm{a}i}(t)$ and its approximation $\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t)$, the state $\tilde{\boldsymbol{x}}_{\mathrm{a}i}$ is reinitialized according to the transformation (11) at the time instants t_{k_i} ($k_i \in \mathbb{N}_0$) which are referred to as event times. In order to determine these event times t_{k_i}, the controller $\mathcal{C}_i$ compares the continuously measured coupling input $\boldsymbol{s}_i(t)$ with the signal $\tilde{\boldsymbol{s}}_i(t)$ produced by the model (15) and triggers an event whenever the condition

$$\|\boldsymbol{s}_i(t) - \tilde{\boldsymbol{s}}_i(t)\| = \bar{e}_i \tag{18}$$

is met, where $\bar{e}_i \in \mathbb{R}_+$ is the event threshold. At the event times

$$\begin{aligned} t_{k_i} &:= \min\{t > t_{k_i-1} \mid \|\boldsymbol{s}_i(t) - \tilde{\boldsymbol{s}}_i(t)\| = \bar{e}_i\}. \\ t_{0_i} &= 0 \end{aligned}$$

the controller $\mathcal{C}_i$ requests the subsystems Σ_j, $j \in \mathcal{N}_i$ to transmit their states $\boldsymbol{x}_j(t_{k_i})$ to $\mathcal{C}_i$. The states $\boldsymbol{x}_j(t_{k_i})$ are then used in (15) in order to reset the model state $\tilde{\boldsymbol{x}}_i$.

In summary, the controller $\mathcal{C}_i$ of subsystem Σ_i continuously measures the subsystem state $\boldsymbol{x}_i(t)$ and the coupling input $\boldsymbol{s}_i(t)$. The state $\boldsymbol{x}_i(t)$ is used to evaluate the model $\tilde{\Sigma}_{\mathrm{a}i}$ given in (15) and, together with the state $\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t)$ for generating the control input (16). Whenever the condition (18) is satisfied the state information $\boldsymbol{x}_j(t_{k_i})$ is requested from all neighboring subsystems Σ_j, ($j \in \mathcal{N}_i$) and is used to reset the state $\tilde{\boldsymbol{x}}_{\mathrm{a}i}(t)$ of the model $\tilde{\Sigma}_{\mathrm{a}i}$. The structure of the controller $\mathcal{C}_i$ is illustrated in Fig. 3.

3.3 Event-based control loop

The extended subsystem $\Sigma_{\mathrm{e}i}$ together with the proposed event-based controller (15), (16), (18) can be formulated as an impulsive system (cf. Donkers and Heemels (2012); Stöcker and Lunze (2013)) that is represented by the model

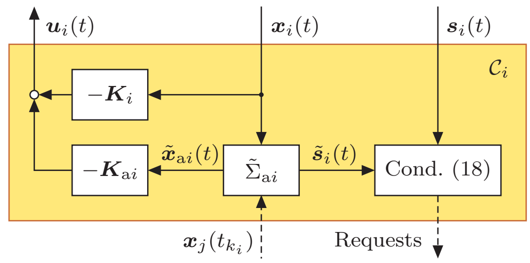

Fig. 3. Structure of the controller $\mathcal{C}_i$

$$\Sigma_{\mathrm{c}i}: \begin{cases} \begin{pmatrix} \dot{\boldsymbol{x}}_{\mathrm{e}i}(t) \\ \dot{\tilde{\boldsymbol{x}}}_{\mathrm{a}i}(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}_{\mathrm{c}i} & \begin{pmatrix} -\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{a}i} \\ \boldsymbol{O} \end{pmatrix} \\ (\boldsymbol{B}_{\mathrm{a}i}\boldsymbol{C}_{\mathrm{z}i} \;\; \boldsymbol{O}) & \boldsymbol{A}_{\mathrm{a}i} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_{\mathrm{e}i}(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{pmatrix} \\ \qquad + \begin{pmatrix} \boldsymbol{E}_{\mathrm{e}i} \\ \boldsymbol{O} \end{pmatrix} \boldsymbol{d}_{\mathrm{e}i}(t) + \begin{pmatrix} \boldsymbol{F}_{\mathrm{e}i} \\ \boldsymbol{O} \end{pmatrix} \boldsymbol{f}_i(t) \\ \begin{pmatrix} \boldsymbol{x}_{\mathrm{e}i}(t_{k_i}^+) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t_{k_i}^+) \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ (\boldsymbol{O} \;\; \boldsymbol{I}) & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_{\mathrm{e}i}(t_{k_i}) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t_{k_i}) \end{pmatrix} \\ \boldsymbol{v}_i(t) = (\boldsymbol{H}_{\mathrm{e}i} \;\; \boldsymbol{O}) \begin{pmatrix} \boldsymbol{x}_{\mathrm{e}i}(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{pmatrix} \end{cases} \tag{19}$$

with

$$\boldsymbol{A}_{\mathrm{c}i} := \begin{pmatrix} \boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_i & \boldsymbol{E}_{\mathrm{s}i}\boldsymbol{C}_{\mathrm{a}i} \\ \boldsymbol{B}_{\mathrm{a}i}\boldsymbol{C}_{\mathrm{z}i} & \boldsymbol{A}_{\mathrm{a}i} \end{pmatrix}. \tag{20}$$

The first equation in (19) is evaluated if

$$\begin{pmatrix} \boldsymbol{x}_{\mathrm{e}i}(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{pmatrix} \in \mathcal{F}_i := \{\boldsymbol{w} \in \mathbb{R}^{n_i+2n_{\mathrm{a}i}} \mid \boldsymbol{w}^\top \boldsymbol{Q}_i \boldsymbol{w} < \bar{e}_i^2\} \tag{21}$$

holds, with

$$\boldsymbol{Q}_i := \begin{pmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{C}_{\mathrm{a}i}^\top\boldsymbol{C}_{\mathrm{a}i} & -\boldsymbol{C}_{\mathrm{a}i}^\top\boldsymbol{C}_{\mathrm{a}i} \\ \boldsymbol{O} & -\boldsymbol{C}_{\mathrm{a}i}^\top\boldsymbol{C}_{\mathrm{a}i} & \boldsymbol{C}_{\mathrm{a}i}^\top\boldsymbol{C}_{\mathrm{a}i} \end{pmatrix}. \tag{22}$$

Accordingly, the state reset in (19) is performed whenever

$$\begin{pmatrix} \boldsymbol{x}_{\mathrm{e}i}(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{pmatrix} \in \mathcal{R}_i := \{\boldsymbol{w} \in \mathbb{R}^{n_i+2n_{\mathrm{a}i}} \mid \boldsymbol{w}^\top \boldsymbol{Q}_i \boldsymbol{w} = \bar{e}_i^2\} \tag{23}$$

is true. The relation between the residual input $\boldsymbol{v}_i(t)$ and the residual output $\boldsymbol{f}_i(t)$ is described by the model (8). The following section investigates the stability of the closed loop system.

4. STABILITY ANALYSIS

This section presents a method for the stability analysis of the control loop (8), (19)–(23) with event-based information requests. For this analysis the system $\Sigma_{\mathrm{c}i}$ is transformed by means of the mappings

$$\boldsymbol{x}_{\mathrm{e}i}(t) = \begin{pmatrix} \boldsymbol{x}_i(t) \\ \boldsymbol{x}_{\mathrm{a}i}(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{I} & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_i(t) \\ \boldsymbol{x}_{\mathrm{a}i}(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{pmatrix}$$

$$\boldsymbol{\delta}_{\mathrm{a}i}(t) = (\boldsymbol{O} \;\; \boldsymbol{I} \;\; -\boldsymbol{I}) \begin{pmatrix} \boldsymbol{x}_i(t) \\ \boldsymbol{x}_{\mathrm{a}i}(t) \\ \tilde{\boldsymbol{x}}_{\mathrm{a}i}(t) \end{pmatrix},$$

which yield the systems

$$\bar{\Sigma}_{\mathrm{e}i}: \begin{cases} \dot{\boldsymbol{x}}_{\mathrm{e}i}(t) = \bar{\boldsymbol{A}}_{\mathrm{e}i}\boldsymbol{x}_{\mathrm{e}i}(t) + \begin{pmatrix} \boldsymbol{B}_i\boldsymbol{K}_{\mathrm{a}i} \\ \boldsymbol{O} \end{pmatrix} \boldsymbol{\delta}_{\mathrm{a}i}(t) \\ \qquad\qquad + \boldsymbol{E}_{\mathrm{e}i}\boldsymbol{d}_{\mathrm{e}i}(t) + \boldsymbol{F}_{\mathrm{e}i}\boldsymbol{f}_i(t) \\ \boldsymbol{x}_{\mathrm{e}i}(t_{k_i}^+) = \boldsymbol{x}_{\mathrm{e}i}(t_{k_i}) \\ \boldsymbol{v}_i(t) = \boldsymbol{H}_{\mathrm{e}i}\boldsymbol{x}_{\mathrm{e}i}(t) \end{cases} \tag{24}$$

where the matrix $\bar{\boldsymbol{A}}_{\mathrm{e}i}$ is defined in (17) and

$$\Delta_{\mathrm{a}i}: \begin{cases} \dot{\boldsymbol{\delta}}_{\mathrm{a}i}(t) = \boldsymbol{A}_{\mathrm{a}i}\boldsymbol{\delta}_{\mathrm{a}i}(t) + \boldsymbol{E}_{\mathrm{a}i}\boldsymbol{d}_{\mathrm{a}i}(t) + \boldsymbol{F}_{\mathrm{a}i}\boldsymbol{f}_i(t) \\ \boldsymbol{\delta}_{\mathrm{a}i}(t_{k_i}^+) = \boldsymbol{0}. \end{cases} \tag{25}$$

The transformed system (24), (25) evolves continuously if $(\boldsymbol{x}_{\mathrm{e}i}^\top \ \boldsymbol{\delta}_{\mathrm{a}i}^\top)^\top \in \mathcal{F}_{\Delta i}$ and the state reset is performed whenever $(\boldsymbol{x}_{\mathrm{e}i}^\top \ \boldsymbol{\delta}_{\mathrm{a}i}^\top)^\top \in \mathcal{R}_{\Delta i}$, where the flow-set $\mathcal{F}_{\Delta i}$ and reset-set $\mathcal{R}_{\Delta i}$ of the transformed system are given by

$$\mathcal{F}_{\Delta i} := \left\{ \boldsymbol{w} \in \mathbb{R}^{n_i + 2n_{\mathrm{a}i}} \mid \boldsymbol{w}^\top \boldsymbol{Q}_{\Delta i} \boldsymbol{w} < \bar{e}_i^2 \right\}, \tag{26a}$$

$$\mathcal{R}_{\Delta i} := \left\{ \boldsymbol{w} \in \mathbb{R}^{n_i + 2n_{\mathrm{a}i}} \mid \boldsymbol{w}^\top \boldsymbol{Q}_{\Delta i} \boldsymbol{w} = \bar{e}_i^2 \right\} \tag{26b}$$

with

$$\boldsymbol{Q}_{\Delta i} := \begin{pmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{C}_{\mathrm{a}i}^\top \boldsymbol{C}_{\mathrm{a}i} \end{pmatrix}. \tag{27}$$

The structure of the transformed overall control loop is illustrated in Fig. 4. Note that the interconnection of $\bar{\Sigma}_{\mathrm{e}i}$ and $\Delta_{\mathrm{a}i}$ is an equivalent representation of $\Sigma_{\mathrm{c}i}$.

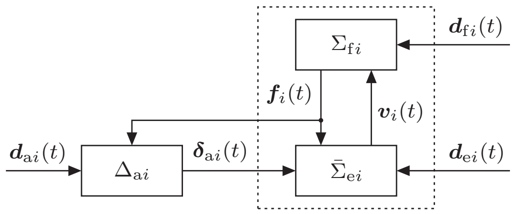

Fig. 4. Structure of the transformed system

Before the main result of this section is formulated in Theorem 8, the following lemma states a sufficient condition for the stability of the system that is composed of $\Sigma_{\mathrm{f}i}$ and $\bar{\Sigma}_{\mathrm{e}i}$ which is marked in Fig. 4 by the dashed frame. The difference state $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ is assumed to satisfy the relation

$$\|\boldsymbol{\delta}_{\mathrm{a}i}(t)\| \le \varepsilon_i, \quad \forall\, t \ge 0 \tag{28}$$

for some finite $\varepsilon_i \in \mathbb{R}_+$.

Lemma 6. Consider the interconnection of the controlled extended subsystem $\bar{\Sigma}_{\mathrm{e}i}$ given in (24) and the residual model $\Sigma_{\mathrm{f}i}$ defined in (8) where the disturbances $\boldsymbol{d}_{\mathrm{f}i}(t)$ and $\boldsymbol{d}_{\mathrm{e}i}(t)$ are bounded as stated in (9) and (13), respectively. Consider $\boldsymbol{f}_i(t)$ to be the output of the interconnection of $\bar{\Sigma}_{\mathrm{e}i}$ and $\Sigma_{\mathrm{f}i}$. Assume that the difference state $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ is bounded according to (28). If $\bar{\Sigma}_{\mathrm{e}i}$ and $\Sigma_{\mathrm{f}i}$ satisfy the relation

$$\int_0^\infty g_{\mathrm{fv}i}(t)\mathrm{d}t \int_0^\infty \left\| \boldsymbol{H}_{\mathrm{e}i} \mathrm{e}^{\bar{\boldsymbol{A}}_{\mathrm{e}i} t} \boldsymbol{F}_{\mathrm{e}i} \right\| \mathrm{d}t < 1, \tag{29}$$

then the interconnection of $\bar{\Sigma}_{\mathrm{e}i}$ and $\Sigma_{\mathrm{f}i}$ is UBIBO-stable.

The proof of Lemma 6 is given in Appendix B. The relation (29) can be interpreted as a small-gain condition which claims that the controlled extended system $\bar{\Sigma}_{\mathrm{e}i}$ and the residual model $\Sigma_{\mathrm{f}i}$ are weakly coupled. In the following, this condition is assumed to be met for all $i \in \mathcal{N}$.

Assumption 7. The systems $\bar{\Sigma}_{\mathrm{e}i}$ and $\Sigma_{\mathrm{f}i}$ satisfy the condition (29) for all $i \in \mathcal{N}$.

Note that the hypothesis of Lemma 6 requires the bound ε_i in (28) to be finite, but the stability condition (29) is independent of the particular magnitude of ε_i. This fact is used in the following to prove the stability of the overall control system (8), (19)–(23) with event-based information requests.

Theorem 8. Consider the control system (8), (19)–(23) where the disturbances $\boldsymbol{d}_{\mathrm{a}i}(t)$, $\boldsymbol{d}_{\mathrm{f}i}(t)$ and $\boldsymbol{d}_{\mathrm{e}i}(t)$ are bounded according to (7), (9) and (13), respectively and let Assumptions 5 and 7 hold for all $i \in \mathcal{N}$. Then the overall control loop (8), (19)–(23) with event-based information requests is ultimately bounded.

Proof. Consider the interconnection of the systems $\bar{\Sigma}_{\mathrm{e}i}$ and $\Sigma_{\mathrm{f}i}$ which satisfy the stability condition (29) by Assumption 7. Hence, the interconnection of these systems is UBIBO-stable if, according to the hypothesis of Lemma 6, the difference state $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ is bounded by some finite bound as stated in (28). In order to see that $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ is bounded for all $t \ge 0$, consider the difference system $\Delta_{\mathrm{a}i}$ as defined in (25) and observe that the output

$$\boldsymbol{C}_{\mathrm{a}i}\boldsymbol{\delta}_{\mathrm{a}i}(t) = \boldsymbol{s}_i(t) - \tilde{\boldsymbol{s}}_i(t)$$

is monitored by the controller $\mathcal{C}_i$ in order to detect the event times t_{k_i}. Recall that an event is triggered whenever the condition

$$\|\boldsymbol{C}_{\mathrm{a}i}\boldsymbol{\delta}_{\mathrm{a}i}(t)\| = \|\boldsymbol{s}_i(t) - \tilde{\boldsymbol{s}}_i(t)\| = \bar{e}_i$$

is met and that the event causes a reset of the difference state $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ to zero (cf. (25)). That is, the relation

$$\|\boldsymbol{C}_{\mathrm{a}i}\boldsymbol{\delta}_{\mathrm{a}i}(t)\| \le \bar{e}_i, \quad \forall\, t \ge 0 \tag{30}$$

holds due to the event triggering and the state reset. Equation (30) together with the observability of the pair $(\boldsymbol{A}_{\mathrm{a}i}, \boldsymbol{C}_{\mathrm{a}i})$ implies the fact that the difference state $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ is bounded for all $t \ge 0$. The previous arguments apply to all $i \in \mathcal{N}$. Hence, the UBIBO-stability of the overall control system (8), (24)–(27) can be inferred from the boundedness of the difference state $\boldsymbol{\delta}_{\mathrm{a}i}(t)$ and the UBIBO-stability of the interconnection of the systems $\bar{\Sigma}_{\mathrm{e}i}$ and $\Sigma_{\mathrm{f}i}$ for all $i \in \mathcal{N}$. Given that (19)–(23) and (24)–(27) are equivalent, the UBIBO-stability of the overall control system (8), (19)–(23) directly follows, which completes the proof. □

5. EXAMPLE

The following example demonstrates the application of the proposed control approach with event-based information requests to a system that consists of $N = 4$ serially interconnected subsystems as given in Appendix A. The subsystems are considered to be of first order and each subsystem is described by the linear state-space model (3) with the parameters

$$\boldsymbol{A}_i = 0.4, \quad \boldsymbol{B}_i = 0.2, \quad \boldsymbol{E}_i = 0.2, \quad \boldsymbol{E}_{\mathrm{s}i} = 0.2, \quad \boldsymbol{C}_{\mathrm{z}i} = 0.2$$
$$\boldsymbol{A}_j = 0.6, \quad \boldsymbol{B}_j = 0.2, \quad \boldsymbol{E}_j = 0.1, \quad \boldsymbol{E}_{\mathrm{s}j} = 0.3, \quad \boldsymbol{C}_{\mathrm{z}j} = 0.4$$

for $i = 1, 2$ and $j = 3, 4$. That is, subsystems Σ_1 and Σ_2 on the one hand and Σ_3 and Σ_4 on the other hand are identical. The interconnection of the subsystems is given by the relations

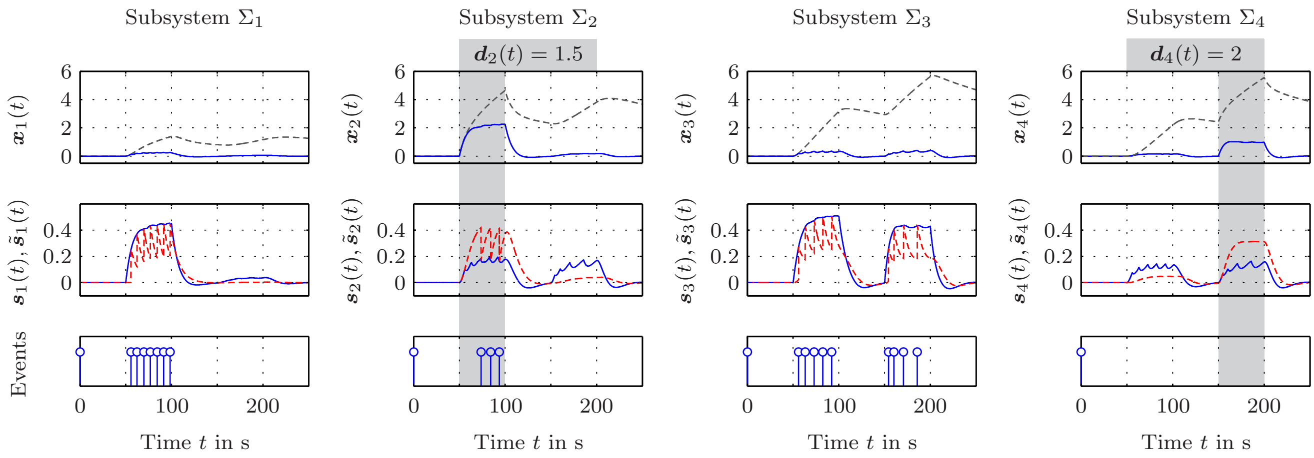

Fig. 5. Disturbance rejection behavior of the control system with event-based information requests

$$\begin{aligned} s_1(t) &= z_2(t), & s_2(t) &= z_1(t) + z_3(t), \\ s_3(t) &= z_2(t) + z_4(t), & z_4(t) &= z_3(t). \end{aligned} \tag{31}$$

The approximate model for each subsystem is designed according to the approach that is explained in Appendix A and the feedback gains are chosen to be

$$\boldsymbol{K}_1 = \boldsymbol{K}_2 = 2.6, \quad \boldsymbol{K}_3 = \boldsymbol{K}_4 = 3.75 \tag{32}$$

and

$$\begin{aligned} \boldsymbol{K}_{a1} &= 0.2, & \boldsymbol{K}_{a2} &= (0.2 \quad 0.4) \\ \boldsymbol{K}_{a3} &= (0.3 \quad 0.6)\,, & \boldsymbol{K}_{a4} &= 0.6 \end{aligned}$$

such that the stability condition (29) holds for all $i \in \mathcal{N} = \{1, 2, 3, 4\}$. In each controller $\mathcal{C}_i$, $i \in \mathcal{N}$ a request for current state information from the neighbor subsystems is triggered if the condition (18) is satisfied where the thresholds are chosen to be

$$\bar{e}_i = 0.25, \quad \forall\, i \in \mathcal{N}. \tag{33}$$

In the following the disturbance rejection behavior of the overall control system with event-based information requests is investigated, where the subsystems Σ_2 and Σ_4 are disturbed:

$$\boldsymbol{d}_2(t) = \begin{cases} 1.5, & \text{if } 50 \le t < 100 \\ 0, & \text{else} \end{cases} \tag{34a}$$

$$\boldsymbol{d}_4(t) = \begin{cases} 2, & \text{if } 150 \le t < 200 \\ 0, & \text{else.} \end{cases} \tag{34b}$$

Although $\boldsymbol{d}_1(t) = \boldsymbol{d}_3(t) = 0$ holds for all $t \ge 0$, the subsystems Σ_1 and Σ_3 are influenced by the disturbances due to the interconnections (31). The subsequently presented simulation results show that, by applying the novel control approach with event-based information requests, the disturbance propagation among the interconnected subsystems is considerably reduced compared to a decentralized continuous state-feedback control.

The simulation results are shown in Fig 5 where the time intervals in which the disturbances (34) are active are highlighted in gray. The top figures illustrate the trajectories of the respective states $\boldsymbol{x}_i(t)$ (solid line). For comparison, these figures also show the state trajectories of the system that is controlled by decentralized continuous state-feedback only (dashed line), where each controller determines the control input according to (16) with (32) and $\boldsymbol{K}_{ai} = \boldsymbol{0}$ for all $i \in \mathcal{N}$. It can be seen that by using decentralized state-feedback only both $\boldsymbol{d}_2(t)$ and $\boldsymbol{d}_4(t)$ have a significant impact on the overall system and not only on the neighbor subsystems as for the novel control approach. This comparison emphasizes that by using event-based information requests the disturbance propagation is considerably suppressed and, therefore, the disturbance rejection behavior of the overall system is improved compared to a decentralized continuous state-feedback control.

The figures in the second row show the coupling input $\boldsymbol{s}_i(t)$ (solid line) and its estimation $\tilde{\boldsymbol{s}}_i(t)$ (dashed line) that is generated by the respective controllers $\mathcal{C}_i$. Whenever both lines deviate by the defined threshold (33), $\mathcal{C}_i$ triggers a request for information from the respective neighbor subsystems which yields a reset of the estimation $\tilde{\boldsymbol{s}}_i(t_{k_i}^+)$ to the current $\boldsymbol{s}_i(t_{k_i})$. The event times are plotted as stems in the bottom figures. In the time interval up to $t = 50$ s the overall system is in steady state and (except the initial events) no information request is triggered. For $t \in [50, 100]$ the subsystem Σ_2 is subject to the disturbance $\boldsymbol{d}_2(t) = 1.5$. Short time after the disturbance gets active it leads to the triggering of events by the controllers $\mathcal{C}_1$ and $\mathcal{C}_3$ of the neighbor subsystems Σ_1 and Σ_3. The influence of the disturbance $\boldsymbol{d}_2(t)$ on Σ_1 and Σ_3 is attenuated due to the event-triggered reinitialization of the approximate models in $\mathcal{C}_1$ and $\mathcal{C}_3$. The effect of the disturbance $\boldsymbol{d}_2(t)$ on subsystem Σ_4 is almost negligible which shows that the disturbance $\boldsymbol{d}_2(t)$ is only marginally propagated over the subsystem Σ_3 such that even no information request is triggered by $\mathcal{C}_4$. This behavior is a characteristic of the proposed control approach with event-based requests which becomes more obvious when investigating the rejection of the disturbance $\boldsymbol{d}_4(t)$. In the time interval $t \in [150, 200]$ the disturbance $\boldsymbol{d}_4(t) = 2$ affects Σ_4 directly and the remaining subsystems via the interconnections. The influence of $\boldsymbol{d}_4(t)$ on Σ_3 causes the triggering of four events and is, hence, sufficiently rejected such that no events are triggered by $\mathcal{C}_1$ and $\mathcal{C}_2$.

6. CONCLUSION

The paper has presented a new distributed control approach for the disturbance rejection in interconnected subsystems. The proposed controller combines local continuous and distributed event-based state feedback where, in contrast to literature, current information is requested rather than sent at the event times. The main analysis results has been stated in Theorem 8 saying that the control system is ultimately bounded if for each subsystem the approximate model, used for the design of each controller, is weakly coupled with the remaining part of the overall system. The state of the approximate model that is incorporated in the controllers is reset at the event times, which are determined based on locally available information only. An illustrative example has shown that by applying the novel control approach with event-based information requests the disturbance propagation is significantly reduced compared to a continuous decentralized state-feedback controller.

REFERENCES

Abraham, R. and Lunze, J. (1992). Modelling and decentralized control of a multizone crystal growth furnance. *Int. J. of Robust and Nonlinear Control*, 2, 107–122.

De Persis, C., Sailer, R., and Wirth, F. (2013). On a small-gain approach to distributed event-triggered control. *Automatica*. To be published.

Dimarogonas, D., Frazzoli, E., and Johansson, K. (2012). Distributed event-triggered control for multi-agent systems. *IEEE Trans. Autom. Control*, 57, 1291–1297.

Donkers, M. and Heemels, W. (2012). Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved decentralised event-triggering. *IEEE Trans. Autom. Control*, 57, 1362–1376.

Hendrick, J.K., Tomizuka, M., and Varaiya, P.P. (1994). Control issues in automated highway systems. *IEEE Control Syst. Mag.*, 14, 21–32.

Khalil, H. (2002). *Nonlinear systems*. Prentice Hall, Upper Saddle River.

Li, L. and Lemmon, M. (2011). Weakly coupled event triggered output feedback control in wireless networked control systems. In *Proc. Allerton Conf. on Communication, Control and Computing*.

Lunze, J. (1992). *Feedback Control of Large-Scale Systems*. Prentice Hall, London.

Lunze, J. and Lehmann, D. (2010). A state-feedback approach to event-based control. *Automatica*, 46, 211–215.

Mazo Jr., M. and Tabuada, P. (2011). Decentralized event-triggered control over wireless sensor/actuator networks. *IEEE Trans. Autom. Control*, 56, 2456–2461.

Seyboth, G., Dimarogonas, D., and Johansson, K. (2013). Event-based broadcasting for multi-agent average consensus. *Automatica*, 49, 245–252.

Sontag, E. (1998). *Mathematical Control Theory: Deterministic Finite Dimensional Systems*. Springer-Verlag, New York.

Stöcker, C. and Lunze, J. (2013). Input-to-state stability of event-based state-feedback control. In *Proc. European Control Conf. 2013*. To be published.

Stöcker, C., Lunze, J., and Vey, D. (2012). Stability analysis of interconnected event-based control loops. In *Proc. Conf. on Analysis and Design of Hybrid Systems*, 58–63.

Wang, X. and Lemmon, M. (2011). Event-triggering in distributed networked control systems. *IEEE Trans. Autom. Control*, 56, 586–601.

Yook, J., Tilbury, D., and Soparkar, N. (2002). Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators. *IEEE Trans. Control Syst. Technol.*, 10, 503–518.

Appendix A. EXAMPLE FOR THE DETERMINATION OF AN APPROXIMATE MODEL

This appendix explains how an approximate model (6) can be determined for the example of $N = 4$ subsystems (3) that are serially interconnected as shown in Fig. 1(a) (cf. Eq. (31)). This coupling structure is representative for several technical systems like a multizone furnace (Abraham and Lunze (1992)) or a platoon of vehicles (Hendrick et al. (1994)).

In the following, an approximate model $\Sigma_{\mathrm{a}2}$ is determined for this system from the viewpoint of Σ_2. For ease of exposition it is assumed that the disturbances $\boldsymbol{d}_i(t)$ are absent for all $i \in \mathcal{N}$. From (31) follows that the subsystems Σ_1, Σ_3 are neighbors of Σ_2 ($\mathcal{N}_2 = \{1, 3\}$). Now consider the subsystems Σ_i ($i = 1, 3, 4$) that are assumed to be controlled by a continuous decentralized state-feedback

$$\boldsymbol{u}_i(t) = -\boldsymbol{K}_i \boldsymbol{x}_i(t)$$

which, hence, yields the controlled subsystems

$$\bar{\Sigma}_i : \begin{cases} \dot{\boldsymbol{x}}_i(t) = \bar{\boldsymbol{A}}_i \boldsymbol{x}_i(t) + \boldsymbol{E}_{\mathrm{s}i} \boldsymbol{s}_i(t), \quad \boldsymbol{x}_i(0) = \boldsymbol{x}_{0i} \\ \boldsymbol{z}_i(t) = \boldsymbol{C}_{\mathrm{z}i} \boldsymbol{x}_i(t) \end{cases}$$

where $\bar{\boldsymbol{A}}_i = (\boldsymbol{A}_i - \boldsymbol{B}_i \boldsymbol{K}_i)$. With the transformation (11)

$$\boldsymbol{x}_{\mathrm{a}2}(t) = \sum_{j \in \mathcal{N}_2} \boldsymbol{T}_{2j} \boldsymbol{x}_j(t) = \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{O} \end{pmatrix} \boldsymbol{x}_1(t) + \begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{I} \end{pmatrix} \boldsymbol{x}_3(t)$$

the approximate model $\Sigma_{\mathrm{a}2}$ is obtained as

$$\Sigma_{\mathrm{a}2} : \begin{cases} \dot{\boldsymbol{x}}_{\mathrm{a}2}(t) = \begin{pmatrix} \bar{\boldsymbol{A}}_1 & \\ & \bar{\boldsymbol{A}}_3 \end{pmatrix} \boldsymbol{x}_{\mathrm{a}2}(t) + \begin{pmatrix} \boldsymbol{E}_{\mathrm{s}1} \\ \boldsymbol{E}_{\mathrm{s}3} \end{pmatrix} \boldsymbol{z}_2(t) \\ \qquad\qquad + \begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{E}_{\mathrm{s}3} \end{pmatrix} \boldsymbol{f}_2(t) \\ \boldsymbol{s}_2(t) = (\boldsymbol{C}_{\mathrm{z}1} \ \boldsymbol{C}_{\mathrm{z}3}) \, \boldsymbol{x}_{\mathrm{a}2}(t) \\ \boldsymbol{v}_2(t) = (\boldsymbol{O} \ \boldsymbol{C}_{\mathrm{z}3}) \, \boldsymbol{x}_{\mathrm{a}2}(t) \end{cases}$$

Figure 1(b) illustrates how the approximate model $\Sigma_{\mathrm{a}2}$ together with Σ_2 form the extended subsystem $\Sigma_{\mathrm{e}2}$ and how $\Sigma_{\mathrm{e}2}$ is interconnected with the residual model $\Sigma_{\mathrm{f}2}$.

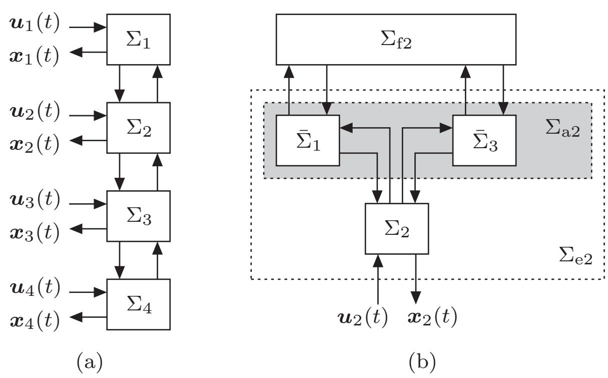

Fig. A.1. Serially interconnected subsystems: (a) structure of the system; (b) decomposition of the system into residual model $\Sigma_{\mathrm{f}2}$, approximate model $\Sigma_{\mathrm{a}2}$ and extended subsystem $\Sigma_{\mathrm{e}2}$ from the viewpoint of Σ_2.

Appendix B. PROOF OF LEMMA 6

For the sake of readability, in the following proof of Lemma 6 the index i is entirely omitted. First, consider the residual system Σ_{f} described by (8). With (10)

$$r_{\mathrm{f}}(t) = g_{\mathrm{fd}} * \hat{d}_{\mathrm{f}} + g_{\mathrm{fv}} * r_{\mathrm{v}} \geq \|\boldsymbol{f}(t)\|\,, \quad \forall\, t \geq 0 \tag{B.1}$$

represents a comparison system for Σ_{f}. Accordingly, the system

$$r_{\mathrm{v}}(t) = g_{\mathrm{v}\varepsilon} * \varepsilon + g_{\mathrm{vd}} * \hat{d}_{\mathrm{e}} + g_{\mathrm{vf}} * r_{\mathrm{f}}(t) \geq \|\boldsymbol{v}(t)\|\,, \quad \forall\, t \geq 0 \tag{B.2}$$

is a comparison system for $\bar{\Sigma}_{\mathrm{e}}$ given in (24) with

$$g_{\mathrm{x}\varepsilon}(t) = \left\|\boldsymbol{H}_{\mathrm{e}} \mathrm{e}^{\bar{\boldsymbol{A}}_{\mathrm{e}} t} \boldsymbol{B}_{\mathrm{e}}\right\|, \quad g_{\mathrm{vd}}(t) = \left\|\boldsymbol{H}_{\mathrm{e}} \mathrm{e}^{\bar{\boldsymbol{A}}_{\mathrm{e}} t} \boldsymbol{E}_{\mathrm{e}}\right\|,$$
$$g_{\mathrm{vf}}(t) = \left\|\boldsymbol{H}_{\mathrm{e}} \mathrm{e}^{\bar{\boldsymbol{A}}_{\mathrm{e}} t} \boldsymbol{F}_{\mathrm{e}}\right\|. \tag{B.3}$$

The substitution of (B.1) in (B.2) yields

$$r_{\mathrm{f}}(t) = g_{\mathrm{fd}} * \hat{d}_{\mathrm{f}} + g_{\mathrm{fv}} * g_{\mathrm{v}\varepsilon} * \varepsilon + g_{\mathrm{fv}} * g_{\mathrm{vd}} * \hat{d}_{\mathrm{e}} + g_{\mathrm{fv}} * g_{\mathrm{vf}} * r_{\mathrm{f}}. \tag{B.4}$$

An explicit bound $r_{\mathrm{f}}(t)$ is obtained from the last equation by means of the comparison principle (Lunze (1992)): Under the condition

$$\int_0^\infty g_{\mathrm{fv}}(t)\mathrm{d}t \int_0^\infty g_{\mathrm{vf}}(t)\mathrm{d}t < 1 \tag{B.5}$$

the impulse response matrices

$$\hat{g}_{\mathrm{f}}(t) = g_{\mathrm{fd}} * \delta(t) + g_{\mathrm{fv}} * g_{\mathrm{vf}} * \hat{g}_{\mathrm{f}}$$
$$\hat{g}_{\varepsilon}(t) = g_{\mathrm{fv}} * g_{\mathrm{v}\varepsilon} * \delta(t) + g_{\mathrm{fv}} * g_{\mathrm{vf}} * \hat{g}_{\varepsilon}$$
$$\hat{g}_{\mathrm{e}}(t) = g_{\mathrm{fv}} * g_{\mathrm{vd}} * \delta(t) + g_{\mathrm{fv}} * g_{\mathrm{vf}} * \hat{g}_{\mathrm{e}}$$

exist and are integrable. Thus, (B.4) can be restated as

$$r_{\mathrm{f}}(t) = \hat{g}_{\mathrm{f}} * \hat{d}_{\mathrm{f}} + \hat{g}_{\varepsilon} * \varepsilon + \hat{g}_{\mathrm{e}} * \hat{d}_{\mathrm{e}}$$

which is known to be UBIBO-stable due to the condition (B.5). Given that $r_{\mathrm{f}}(t)$ is bounded for all $t \geq 0$, from (B.2) the boundedness of $\|\boldsymbol{f}(t)\|$ for all $t \geq 0$ follows. Finally, observe that (B.5) with (B.3) is equivalent to (29) which completes the proof of Lemma 6. □